THE METHOD OF MONOTONE ITERATIONS FOR MIXED MONOTONE OPERATORS IN PARTIALLY ORDERED SETS AND ORDER-ATTRACTIVE FIXED POINTS

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Abstract. We use the method of monotone iterations to obtain fixed point and coupled fixed point results for mixed monotone operators in the setting of partially ordered sets, with no additional assumptions on the partial order and with no convergence structure. We define the concept of attractive fixed point with respect to the partial order and obtain several criteria for the existence, uniqueness and order-attractiveness of the fixed points, both in the presence and in the absence of a coupled lower-upper fixed point. As an application, we present a fixed point result for a class of mixed monotone operators in the setting of ordered linear spaces.

Key Words and Phrases: Partially ordered set, mixed monotone operator, monotone iterative method, fixed point, coupled fixed point, coupled lower-upper fixed point, order-attractive point, ordered linear space, cone.

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