

## APPROXIMATING FIXED POINTS OF 2-GENERALIZED HYBRID MAPPINGS IN BANACH SPACES AND CAT(0) SPACES

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**Abstract.** In this paper, we first prove weak and strong convergence theorems for Ishikawa and Halpern iterations of 2-generalized hybrid mappings in uniformly convex Banach spaces and we apply our method to provide an affirmative answer to an open problem raised by Hojo, Takahashi and Termwuttipong [Strong convergence theorems for 2-generalized hybrid mappings in Hilbert spaces, *Nonlinear Analysis*, 75 (2012) 2166-2176]. We then extend the results to CAT(0) spaces, which include especially simply connected complete Riemannian manifolds with nonpositive sectional curvature. Our results improve and generalize some known results in the current literature.

**Key Words and Phrases:** 2-generalized hybrid mapping, fixed point, uniformly convex Banach space, CAT(0) spaces, Riemannian manifolds, weak convergence, strong convergence.

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