

## THE FIXED POINT ALTERNATIVE THEOREM AND SET-VALUED FUNCTIONAL EQUATIONS

ALIREZA KAMEL MIRMOSTAFAEI AND MOSTAFA MAHDAVI

Center of Excellence in Analysis on Algebraic Structures  
Department of Pure Mathematics, School of Mathematical Sciences  
Ferdowsi University of Mashhad  
Mashhad 91775, Iran  
E-mail: mirmostafaei@ferdowsi.um.ac.ir

**Abstract.** We use the fixed point alternative theorem to prove the stability of the set-valued function equation

$$c(x)F(h(x)) = F(x).$$

This result enable us to prove the stability of some set-valued functional equations.

**Key Words and Phrases:** Set-valued mappings, functional inequalities, non-expensive mappings, fixed point.

**2010 Mathematics Subject Classification:** 39B52, 39B69, 47S09, 47H10.

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*Received: May 02, 2012; Accepted: May 05, 2013*