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PERIODIC SOLUTIONS OF SECOND ORDER NON-AUTONOMOUS DIFFERENTIAL SYSTEMS

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Abstract. We study the existence of nonnegative solutions for second order nonlinear differential systems with periodic boundary conditions. In this class of problems, where the associated Green's function may take on negative values, and the nonlinear term is allowed to be singular. Our method is based on the Guo-Krasnosel'skii fixed point theorem of cone expansion and compression type, involving a new type of cone. Recent results in the literature, even in the scalar case, are complemented, generalized and improved.
Key Words and Phrases: Nonnegative solutions, existence, Guo-Krasnosel'skii fixed point theorem, differential systems.
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