

ON NONLOCAL PROBLEMS FOR RETARDED FRACTIONAL DIFFERENTIAL EQUATIONS IN BANACH SPACES

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Abstract. We study the existence and asymptotic stability of solutions for a class of Cauchy problems involving retarded semilinear fractional differential equations subject to nonlocal conditions. The results are proved by means of fractional calculus and fixed point theory for condensing maps. They in particular extend and improve many recent existence results for fractional differential equations. An example is also given to illustrate the results.

Key Words and Phrases: Asymptotic stability; fractional differential equation; finite delay; non-local condition; condensing map; fixed point; measure of non-compactness; MNC-estimate.

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