

STRONG CONVERGENCE THEOREM FOR INFINITE FAMILY OF TOTAL QUASI- ϕ -ASYMPTOTICALLY NONEXPANSIVE MULTI-VALUED MAPPINGS

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Abstract. We prove strong convergence theorem for infinite family of uniformly L -Lipschitzian total quasi- ϕ -asymptotically nonexpansive multi-valued mappings using a generalized f -projection operator in a real uniformly convex and uniformly smooth Banach space. The result presented in this paper improve and unify important recent results announced by many authors.

Key Words and Phrases: total quasi- ϕ -asymptotically nonexpansive multi-valued mapping, fixed point, strong convergence, f -projection operator.

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