SOLVABILITY OF A FUNCTIONAL EQUATION ARISING
IN DYNAMIC PROGRAMMING

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Abstract. This paper deals with solvability of the following functional equation arising in dynamic
programming of multistage decision processes

\[ f(x) = \text{opt}_{y \in D} \{ u(x, y)(p(x, y) + f(a(x, y))) + v(x, y) \text{opt}_{q(x, y)} \{ q(x, y), f(b(x, y)) \} \}, \quad \forall x \in S. \]

Using the Banach fixed point theorem and new iterative techniques, we obtain the existence and
uniqueness of solutions for the above equation in the complete metric space \( BB(S) \) and the Banach
spaces \( BC(S) \) and \( B(S) \), construct some iterative methods, prove their convergence and provide
several error estimates between these iterative sequences generated by the iterative methods and
the corresponding solutions, respectively. Four nontrivial examples illustrating applications of the
results presented in this paper are provided.

Key Words and Phrases: Dynamic programming, functional equation, Banach fixed point theo-
rem, nonexpansive mapping, iterative methods, error estimates.

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