

## ON THE FIXED POINT THEOREMS OF CARISTI TYPE

GUOWEI ZHANG AND DAN JIANG

Department of Mathematics, Northeastern University  
Shenyang 110819, China  
E-mail: gwzhangneum@sina.com

**Abstract.** In this paper, using the minimal element method, we give some fixed point theorems of Caristi type which extend the previous results due to Amini-Harandi [Nonlinear Anal. 72 (2010) 4661–4665], Khamsi [Nonlinear Anal. 71 (2009) 227–231], Suzuki [J. Math. Anal. Appl. 302 (2005) 502–508] and others. Moreover, variational theorem of Ekeland type is discussed.

**Key Words and Phrases:** Caristi's fixed point, Ekeland's variational principle, minimal element, partially ordered set

**2010 Mathematics Subject Classification:** 47H10, 54H25

**Acknowledgements.** The authors express their gratitude to the referees for their valuable comments that make the proof of the main theorem more concise.

### REFERENCES

- [1] J. Caristi, *Fixed point theorems for mappings satisfying inwardness conditions*, Trans. Amer. Math. Soc., **215**(1976), 241-251.
- [2] A. Brøndsted, *Fixed points and partial orders*, Proc. Amer. Math. Soc., **60**(1976), 365-366.
- [3] J. Caristi, W.A. Kirk, *Geometric fixed point theory and inwardness conditions*, in: Lecture Notes in Math., vol. 490, Springer-Verlag, New York, 1975, 74–83.
- [4] I. Ekeland, *On the variational principle*, J. Math. Anal. Appl., **47**(1974), 324-353.
- [5] I. Ekeland, *Nonconvex minimization problems*, Bull. Amer. Math. Soc., **1**(1979), 443-474.
- [6] S. Shi, *Equivalence between Ekeland's variational principle and Caristi's fixed point theorem*, Advan. Math., **16**(1987), 203-206 (in Chinese).
- [7] R.P. Agarwal, M.A. Khamsi, *Extension of Caristi's fixed point theorem to vector valued metric spaces*, Nonlinear Anal., **74**(2011), 141-145.
- [8] A. Amini-Harandi, *Some generalizations of Caristi's fixed point theorem with applications to the fixed point theory of weakly contractive set-valued maps and the minimization problem*, Nonlinear Anal., **72**(2010), 4661-4665.
- [9] J.S. Bae, *Fixed point theorems for weakly contractive multivalued maps*, J. Math. Anal. Appl., **284**(2003), 690-697.
- [10] J.S. Bae, E.W. Cho, S.H. Yeom, *A generalization of the Caristi-Kirk fixed point theorem and its applications to mapping theorems*, J. Korean Math. Soc., **31**(1994), 29-48.
- [11] D. Downing, W.A. Kirk, *A generalization of Caristi's theorem with applications to nonlinear mapping theory*, Pacific J. Math., **69**(1977), 339-346.
- [12] Y. Feng, S. Liu, *Fixed point theorems for multi-valued contractive mappings and multi-valued Caristi type mapping*, J. Math. Anal. Appl., **317**(2006), 103-112.
- [13] M.A. Khamsi, *Remarks on Caristi's fixed point theorem*, Nonlinear Anal., **71**(2009), 227-231.

- [14] A. Latif, *Generalized Caristi's fixed point theorems*, Fixed Point Theory Appl., **2009**(2009) Article ID 170140, 7 pages.
- [15] Z. Li, *Remarks on Caristi's fixed point theorem and Kirk's problem*, Nonlinear Anal., **73**(2010), 3751-3755.
- [16] J.L. Sun, J.X. Sun, *A generalization of Caristi's fixed point theorem and its applications*, J. Math. Res. Exposition, **26**(2006), 199-206.
- [17] T. Suzuki, *Generalized Caristi's fixed point theorems by Bae and others*, J. Math. Anal. Appl., **302**(2005), 502-508.
- [18] T. Suzuki, *Generalized distance and existence theorems in complete metric spaces*, J. Math. Anal. Appl., **253**(2001), 440-458.
- [19] A. Hamel, *Variational Principles on Metric and Uniform Spaces*, Habilitation Thesis, Halle, 2005.

*Received: November 2, 2011; Accepted: April 26, 2012.*

