

## FIRST ORDER MULTIVALUED PROBLEMS ON TIME SCALES

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**Abstract.** We present an existence result for first order dynamic inclusions on time scales with a periodic boundary value condition, assuming the existence of a solution tube for the considered inclusion and the Henstock integrability of the multifunction on the right-hand side.

**Key Words and Phrases:** dynamic inclusion, solution tube, Henstock integral.

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