

## FIXED POINT THEOREMS FOR NONCONVEX VALUED CORRESPONDENCES AND APPLICATIONS IN GAME THEORY

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**Abstract.** In this paper, we introduce several types of correspondences: weakly naturally quasiconvex, \*-weakly naturally quasiconvex, weakly biconvex and correspondences with \*-weakly convex graph and we prove some fixed point theorems for these kinds of correspondences. As a consequence, using a version of W.K. Kim's quasi-point theorem, we obtain the existence of equilibria for a quasi-game.

**Key Words and Phrases:** Fixed point theorem, correspondences with \*-weakly convex graph, weakly naturally quasiconvex correspondences, quasi game, quasi-equilibrium.

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