

## A HALPERN-LIONS-REICH-LIKE ITERATIVE METHOD FOR NONEXPANSIVE MAPPINGS

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**Abstract.** We prove strong convergence of a Halpern-Lions-Reich-like iterative algorithm for approximating fixed points of nonexpansive mappings in a uniformly smooth Banach space. The idea of this algorithm is then applied to solve a quadratic minimization problem in a Hilbert space.

**Key Words and Phrases:** Halpern-Lions-Reich-like iterative algorithm, nonexpansive mapping, fixed point, uniformly smooth Banach space, quadratic minimization problem.

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