

ON THE SET OF SOLUTIONS FOR THE DARBOUX PROBLEM FOR FRACTIONAL ORDER PARTIAL HYPERBOLIC FUNCTIONAL DIFFERENTIAL INCLUSIONS

SÄID ABBAS* AND MOUFFAK BENCHOHRA**

*2320, Rue de Salaberry, apt 10
Montréal, QC H3M 1K9, Canada
E-mail: abbasmsaid@yahoo.fr

**Laboratoire de Mathématiques, Université de Sidi Bel-Abbès,
B.P. 89, 22000, Sidi Bel-Abbès, Algérie
E-mail: benchohra@univ-sba.dz

Abstract. In this paper we prove the arcwise connectedness of the solution set for the initial value problems (IVP for short), for a class of nonconvex and nonclosed functional hyperbolic differential inclusions of fractional order.

Key Words and Phrases: Partial hyperbolic differential inclusion; fractional order; left-sided mixed Riemann-Liouville integral; Caputo fractional derivative; solution set, arcwise connected.

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