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FRACTIONAL EQUATIONS AND A THEOREM OF BROUWER-SCHAUDER TYPE

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Abstract. Brouwer's fixed point theorem states that a continuous mapping of a closed, bounded, convex, nonempty set $M \subset R^n$ into itself has a fixed point. Schauder's theorem states that a continuous mapping of a closed, convex, nonempty set M in a Banach space has a fixed point, provided that M is mapped into a compact subset of itself. In this brief note we point out that for a large class of differential equations which are transformed into integral equations defining the mapping, then that last compactness condition can be dropped, provided that M is bounded in the supremum norm. The set M is usually composed of continuous functions $\phi : [0, \infty) \to \Re$ and it can be a substantial task to prove compactness, sometimes requiring draconian conditions such as all the functions in M having the same limit at ∞. In effect, then, we reduce the conditions of Schauder's theorm (in function spaces with domains on an infinite interval) to the conditions of the far simpler Brouwer's theorem in R^n for this class of problems.
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