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THE METHOD OF SUCCESSIVE INTERPOLATIONS SOLVING INITIAL VALUE PROBLEMS FOR SECOND ORDER FUNCTIONAL DIFFERENTIAL EQUATIONS

A. M. BICA*, M. CURILĂ** AND S. CURILĂ***

*Department of Mathematics and Informatics,
University of Oradea, Universitatii Street no.1, 410087 Oradea, Romania
E-mail: abica@uoradea.ro

**Faculty of Environment Protection,
University of Oradea, Universitatii Street no.1, 410087 Oradea, Romania

***Faculty of Electric Engineering and Information Technology,
University of Oradea, Universitatii Street no.1, 410087 Oradea, Romania

Abstract. A new numerical method for initial value problems associated to second order functional differential equations is obtained. The method uses the fixed point technique, the trapezoidal quadrature rule, and a Birkhoff interpolation procedure. The convergence of the method is proved without smoothness conditions, the kernel function being only Lipschitzian in each argument. The interpolation procedure is used only on the points where the argument is modified. A stopping criterion of the algorithm is obtained and the accurancy of the method is illustrated on some numerical examples of pantograph type.
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