DENSITY OF THE SET OF RENORMINGS IN c_0 WITHOUT ASYMPTOTICALLY ISOMETRIC COPIES OF c_0 AND FAILING TO HAVE THE FIXED POINT PROPERTY

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*Dedicated to K. Goebel on the occasion of his retirement, and to L. Ciric, W.A. Kirk
and I. A. Rus on the occasion of their 75th birthday*

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Abstract. It is proved that the family of all equivalent norms in c_0 without asymptotically isometric copies of c_0 and failing to have the fixed point property is dense in the set of all renormings of c_0.
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REFERENCES

[1] T. Domínguez Benavides, *Distortion and stability of the fixed point property for non-expansive mappings*, Nonlinear Anal., **75**(2012), 3229-3234.

[2] P.N. Dowling, C.J. Lennard, B. Turett, *Asymptotically perturbed norms of classical sequence spaces with applications to fixed point theory,* Proceedings of Workshop on Fixed Point Theory (Kazimierz Dolny, 1997), Ann. Univ. Mariae Curie-Sklodowska Sect. A, **51**(1997), no. 2, 67–98.

[3] P.N. Dowling, C.J. Lennard, *Every nonreflexive subspace of $L_1[0,1]$ fails the fixed point property*, Proc. Amer. Math. Soc., **125**(1997), 443-446.

[4] P.N. Dowling, C.J. Lennard, B. Turett, *Asymptotically Isometric Copies of c_0 in Banach Spaces*, J. Math. Anal. Appl., **219**(1998), 377-391.

[5] P.N. Dowling, P.K. Lin, B. Turett, *Failure of the FPP inside an asymptotically isometric-free copy of c_0,* Nonlinear Anal., **73**(2010), 1175-1179.

[6] P.N. Dowling, N. Randrianantoanina, *Spaces of compact operators on a Hilbert space with the fixed point property*, J. Funct. Anal., **168**(1)(1999), 111-120.

[7] M. Fabian, L. Zajicek, V. Zizler, *On residuality of the set of rotund norms on a Banach space*, Math. Ann., **258**(1982), 349-351.

[8] K. Goebel, W. Kirk, *Topics in Metric Fixed Point Theory*, Cambridge University Press, 1990.

[9] C. Hernández-Linares, M. Japón, C. Lennard, *Renormings failing to have asymptotically isometric copies of ℓ_1 or c_0,* Nonlinear Anal. (to appear) http://dx.doi.org/10.1016/j.na.2012.09.006.

[10] W.A. Kirk, B. Sims (Ed.), *Handbook of Metric Fixed Point Theory*, Kluwer Academic Publishers, 2001.

[11] R.C. James, *Uniformly non-square Banach spaces,* Ann. of Math., **80**(1964), no. 2, 542-550.

[12] P.K. Lin, *There is an equivalent norm on ℓ_1 that has the fixed point property*, Nonlinear Anal., **68**(2008), no. 8, 2303-2308.

[13] H. Pfitzner, *A note on asymptotically isometric copies of ℓ_1 and c_0*, Proc. Amer. Math. Soc., **129**(5)(2001), 1367-1373.