

# EXISTENCE AND MULTIPLICITY OF POSITIVE SOLUTIONS FOR SECOND-ORDER SELF-ADJOINT BOUNDARY VALUE PROBLEM WITH INTEGRAL BOUNDARY CONDITIONS AT RESONANCE

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**Abstract.** In this paper, we are concerned with the second order self-adjoint boundary value problem at resonance

$$\begin{aligned} -(p(t)x'(t))' &= f(t, x(t)), \quad t \in (0, 1), \\ x'(0) &= 0, \quad x(1) = \int_0^1 x(s)g(s)ds. \end{aligned}$$

A few new results are given for the existence of at least one, two, three and  $n$  positive solutions of the above boundary value problem by using the theory of a fixed point index for A-proper semilinear operators defined on cones, where  $n$  is an arbitrary natural number.

**Key Words and Phrases:** Boundary value problem, positive solution, resonance, multiplicity, A-proper, fixed point index.

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