

NONLOCAL INITIAL VALUE PROBLEMS FOR FIRST ORDER DIFFERENTIAL SYSTEMS

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Abstract. The paper is devoted to existence of solutions to initial value problems for nonlinear first order differential systems with nonlocal conditions. The proof will rely on the Perov, Schauder and Leray-Schauder fixed point principles which are applied to a nonlinear integral operator. The novelty in this paper is that this approach is combined with the technique that uses convergent to zero matrices and vector norms.

Key Words and Phrases: Nonlinear differential system, nonlocal initial condition, fixed point, vector norm, matrix convergent to zero.

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