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NON-SMOOTH GUIDING FUNCTIONS AND PERIODIC SOLUTIONS OF FUNCTIONAL DIFFERENTIAL INCLUSIONS WITH INFINITE DELAY IN HILBERT SPACES

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Abstract. In this paper we develop the method of non-smooth integral guiding functions to deal with the problem of existence of periodic solutions for functional differential inclusions with infinite delay in Hilbert spaces. As an example we study the periodic problem for a gradient functional differential inclusion with infinite delay.
Key Words and Phrases: Non-smooth guiding function, integral guiding function, functional differential inclusion, infinite delay, periodic solution, topological degree.
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References

[1] V. Barbu, *Nonlinear Semigroups and Differential Equations in Banach Spaces*, Noordhoff International Publishing, Leyden, 1976.
[2] Y.G. Borisovich, B.D. Gelman, A.D. Myshkis, V.V. Obukhovskii, *Introduction to the Theory of Multivalued Maps and Differential Inclusions*, Librokom, Moscow, 2011 (in Russian).
[3] F.H. Clarke, *Optimization and Nonsmooth Analysis,* Second edition, Classics in Applied Mathematics 5, Society for Industrial and Applied Mathematics (SIAM), Philadelphia, PA, 1990.
[4] K. Deimling, *Nonlinear Functional Analysis*, Springer-Verlag, Berlin, 1985.
[5] V.F. Dem'yanov, L.V. Vasil'ev, *Nondifferentiable Optimization.* Nauka, Moscow 1981 (in Russian); English translation: Translation Series in Mathematics and Engineering. Optimization Software, Inc., Publications Division, New York, 1985.
[6] I. Ekeland, R. Temam, *Convex Analysis and Variation Problems*, North Holland, Amsterdam, 1979.

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[7] A. Fonda, *Guiding functions and periodic solutions to functional differential equations*, Proc. Amer. Math. Soc., **99**(1987), no. 1, 79-85.

[8] R.E. Gaines, J.L. Mawhin, *Coincidence Degree and Nonlinear Differential Equations*, Lecture Notes in Mathematics, no. 568, Springer-Verlag, Berlin-New York, 1977.

[9] L. Górniewicz, *Topological Fixed Point Theory of Multivalued Mappings.* Second edition. Topological Fixed Point Theory and Its Applications, 4. Springer, Dordrecht, 2006.

[10] J.K. Hale, J. Kato, *Phase space for retarded equations with infinite delay*, Funkcial. Ekvac., **21**(1978), no. 1, 11-41.

[11] Y. Hino, S. Murakami, T. Naito, *Functional Differential Equations with Infinite Delay.* Lecture Notes in Mathematics, Vol. 1473, Springer-Verlag, Berlin–Heidelberg–New York, 1991.

[12] M. Kamenskii, V. Obukhovskii, P. Zecca, *Condensing Multivalued Maps and Semilinear Differential Inclusions in Banach Spaces*, de Gruyter Series in Nonlinear Analysis and Applications 7, Walter de Gruyter, Berlin-New York 2001.

[13] S.V. Kornev, V.V. Obukhovskii, *On non-smooth multivalent guiding functions*, Differ. Uravn., **39**(2003), no. 11, 1497-1502 (in Russian); English translation: Differ. Equ., **39**(2003), no. 11, 1578-1584.

[14] S. Kornev, V. Obukhovskii, *On some developments of the method of integral guiding functions*, Funct. Differ. Equ., **12**(2005), no. 3-4, 303-310.

[15] S.V. Kornev, V.V. Obukhovskii, *Nonsmooth guiding potentials in problems of forced oscillations*, Avtomat. i Telemekh., 2007, no. 1, 3-10 (in Russian); English translation: Automation Remote Control, **68**(2007), no. 1, 1-8.

[16] M.A. Krasnosel'skii, *The Operator of Translation Along the Trajectories of Differential Equations*, Nauka, Moscow 1966 (in Russian); English translation: Translations of Mathematical Monographs 19, Amer. Math. Soc., Providence, R.I. 1968.

[17] M.A. Krasnosel'skii, A.I. Perov, *On a certain priciple of existence of bounded, periodic and almost periodic solutions of systems of ordinary differential equations*, Dokl. Akad. Nauk SSSR, **123**(1958), no. 2, 235-238 (in Russian).

[18] M.A. Krasnosel'skii, P.P. Zabreiko, *Geometrical Methods of Nonlinear Analysis*, Nauka, Moscow 1975; English translation: Grundlehren der Mathematischen Wissenschaften 263, Springer-Verlag, Berlin, 1984.

[19] Nguyen Van Loi, *Method of guiding functions for differential inclusions in a Hilbert space*, Differ. Uravn., **46**(2010), no. 10, 1433-1443 (in Russian); English translation: Differ. Equ., **46**(2010), no. 10, 1438-1447.

[20] L. Schwartz, *Cours d'Analyse*, 1, Second edition. Hermann, Paris, 1981.

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