MINIMUM-NORM FIXED POINT OF NONEXPANSIVE NONSELF MAPPINGS IN HILBERT SPACES

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Abstract. Both implicit and explicit methods are introduced to find the minimum-norm fixed point of a nonexpansive nonself mapping from a closed convex subset C of a Hilbert space H into H and satisfying the weak inwardness condition. Our idea is to apply the nearest point projection P_C to the well-known Browder's implicit and Halpern's explicit methods.

Key Words and Phrases: Nonexpansive nonself mapping, nearest point projection, fixed point, minimum-norm, Browder's method, Halpern's method, weak inwardness condition.

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