

CONVERGENCE OF MODIFIED ISHIKAWA ITERATIVE PROCESSES FOR AN INFINITE FAMILY OF NONEXPANSIVE MAPPINGS

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Abstract. The purpose of this article is to modify Ishikawa iterative process to have strong convergence for an infinite family nonexpansive mappings. Convergence theorems are established in a real Banach space. The results presented in this paper mainly improve the corresponding results announced in [3], [15] and [31].

Key Words and Phrases: Nonexpansive mapping, fixed point, strong convergence, control sequence, Ishikawa iterative process.

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