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A DUALITY FIXED POINT THEOREM AND APPLICATIONS

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Abstract. Let E be a $2-$uniformly convex Banach space with the $2-$uniformly convex constant $1/c$, let $T : E \to E^*$ be a $L-$Lipschitz mapping with condition $0 < \frac{2L}{c^2} < 1$. Then T has a unique duality fixed point $x^* \in E$ $(Tx^* = Jx^*)$ and for any given guess $x_0 \in E$, the iterative sequence $x_{n+1} = J^{-1}Tx_n$ converges strongly to this duality fixed point x^*. If $0 < \frac{2L}{c^2} \leq 1$ and the duality fixed point set of T is nonempty, let $\{\alpha_n\} \subset [0, 1]$ be a real sequence which satisfies the condition $\sum_{n=0}^{\infty} \alpha_n(1 - \alpha_n) = +\infty$, then for any guess $x_0 \in E$, the iterative sequence $x_{n+1} = (1 - \alpha_n)x_n + \alpha_n J^{-1}Tx_n$ converges weakly to a duality fixed point. This main result can be used for solving the variational inequalities and optimal problems.
Key Words and Phrases: 2-uniformly smooth Banach space, dual space, fixed point, contraction mapping principle, application.
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