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NOVEL COMPUTATIONAL DERIVATIVE-FREE METHODS FOR SIMPLE ROOTS

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Abstract. Some novel computational techniques for solving single variable nonlinear equations are given. The schemes are without memory and free from derivative evaluations per full iteration. They are built by applying the weight function approach alongside an approximation for the first derivative of the function in the second step of a two-step cycle for obtaining optimal fourth-order schemes; and also by adapting a nonlinear fraction in the third-step of a three-step cycle to attain seventh-order techniques. The classical efficiency indices of the proposed two- and three-step derivative-free methods are 1.587 and 1.626, respectively up to now. Further research has also been done via the concept of weight functions to provide optimal eighth-order derivative-free techniques which possess 1.682 as their efficiency index. The superiority of the techniques is illustrated by solving numerical examples.

Key Words and Phrases: Nonlinear equations, efficiency index, optimality, simple root, two-step iterative methods, three-step iterative methods, derivative-free, convergence rate.
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References

[1] M.T. Darvishi, *Some three-step iterative methods free from second order derivative for finding solutions of systems of nonlinear equations*, Int. J. Pure Appl. Math., **57**(2009), 557–574.
[2] Y.H. Geum, Y.I. Kim, *A biparametric family of eighth-order methods with their third-step weighting function decomposed into a one-variable linear fraction and a two-variable generic function*, Comput. Math. Appl., **61**(2011), 708–714.
[3] A. Iliev, N. Kyurkchiev, *Nontrivial Methods in Numerical Analysis: Selected Topics in Numerical Analysis*, LAP Lambert Academic Publishing, 2010.
[4] H.T. Kung, J.F. Traub, *Optimal order of one-point and multipoint iteration*, J. ACM, **21**(1974), 643–651.
[5] Y. Lu, X. Xu, *A new family of modified Newton methods with cubic convergence*, Fixed Point Theory, **8**(2007), 47–57.
[6] I. Pavaloiu, *Bilateral approximations of solutions of equations by order three Steffensen-type methods*, Stud. Univ. Babes-Bolyai, Math., **51**(2006), 105–114.
[7] M. Rafiullah, M. Haleem, *Three-step iterative method with sixth order convergence for solving nonlinear equations*, Int. J. Math. Anal., **4**(2010), 2459–2463.

[8] F. Soleymani, M. Sharifi, *On a cubically iterative scheme for solving nonlinear equations*, Far East J. Appl. Math., **43**(2010), 137–143.
[9] F. Soleymani, *Revisit of Jarratt method for solving nonlinear equations*, Numer. Algorithms, (2010), doi: 10.1007/s11075-010-9433-6.
[10] F. Soleymani, *Regarding the accuracy of optimal eighth-order methods*, Math. Comput. Modelling, **53**(2011), 1351–1357.
[11] F. Soleymani, *Concerning some sixth-order iterative methods for finding the simple roots of nonlinear equations*, Bull. Math. Anal. Appl., **2**(2010), 146–151.
[12] J.F. Traub, *Iterative Methods for the Solution of Equations*, Prentice Hall, 1964.
[13] Q. Ye, X. Xu, *A class of Newton-like methods with cubic convergence for nonlinear equations*, Fixed Point Theory, **11**(2010), 161–168.
[14] M.Y. Waziri, W.J. Leong, M.A. Hassan, M. Monsi, *An efficient solver for systems of nonlinear equations with singular Jacobian via diagonal updating*, Appl. Math. Sci., **4**(2010), 3403–3412.

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