

## CONVERGENCE THEOREMS FOR APPROXIMATION OF FIXED POINTS OF NONEXPANSIVE MAPPINGS IN BANACH SPACES

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**Abstract.** Let  $E$  be a uniformly smooth and uniformly convex real Banach space and let  $K$  be a nonempty, closed and convex sunny nonexpansive retract of  $E$  with  $Q_K$  as the sunny nonexpansive retraction. Let  $T : K \rightarrow K$  be a nonexpansive mapping such that  $F(T) \neq \emptyset$ . Assume that either  $E$  admits weakly sequentially continuous duality mapping  $j$  or  $T$  is demicompact. Then, we introduce two approximation schemes (implicit and explicit) for finding a fixed point of a nonexpansive mapping and prove strong convergence of the schemes. Our results extend the recent results of Yao *et al.* [Strong convergence of two iterative algorithms for nonexpansive mappings in Hilbert spaces, *Fixed Point Theory Appl.* volume 2009 (2009), Article ID 279058, 7 pages].

**Key Words and Phrases:** Strong convergence, nonexpansive mappings, uniformly smooth spaces, uniformly convex spaces.

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