

## AN ABSTRACT POINT OF VIEW ON ITERATIVE APPROXIMATION OF FIXED POINTS: IMPACT ON THE THEORY OF FIXED POINT EQUATIONS

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**Abstract.** Let  $(X, \rightarrow)$  be an  $L$ -space,  $G : X \times X \rightarrow X$  and  $f : X \rightarrow X$  be two operators. Let  $f_G : X \rightarrow X$  be defined by  $f_G(x) := G(x, f(x))$ . If the operator  $G$  satisfies the following conditions:

- (A<sub>1</sub>)  $G(x, x) = x, \forall x \in X$ ;
- (A<sub>2</sub>)  $G(x, y) = x \Rightarrow y = x$ ,

then we call  $f_G$  admissible perturbation of  $f$ .

We introduce some iterative algorithms in terms of admissible perturbations. We suppose that these algorithms are convergent.

In this paper we study the impact of this hypothesis on the theory of fixed point equations: Gronwall lemmas (when  $(X, \rightarrow, \leq)$  is an ordered  $L$ -space), data dependence, stability and shadowing property (when  $(X, d)$  is a metric space). Some open problems are presented.

**Key Words and Phrases:** fixed point, admissible perturbation, iterative method, Gronwall lemma, comparison lemma, data dependence, stability, Ulam-Hyers stability, limit shadowing property, open problem.

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