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HYBRID-EXTRAGRADIENT TYPE METHODS FOR A GENERALIZED EQUILIBRIUM PROBLEM AND VARIATIONAL INEQUALITY PROBLEMS OF NONEXPANSIVE SEMIGROUPS

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Abstract. We study and introduce modified mann iterative algorithms for finding a common element of the set of solutions of a generalized equilibrium problem, the set of solutions of variational inequalities and the set of fixed points for nonexpansive semigroups. Then, we prove strong convergence theorems in a real Hilbert space by using the hybrid-extragradient type methods in the mathematical programming under some appropriate control conditions.
Key Words and Phrases: Generalized equilibrium problem, variational inequalities, Strong convergence, Nonexpansive, Semigroup, Hilbert space, Extragradient method, Hybrid method.
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