

INTEGRO-DIFFERENTIAL EQUATION WITH TWO TIME LAGS

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Abstract. We consider an integro-differential equation with two time lags and we prove the existence, uniqueness and convergence of the sequence of the successive approximation by using contraction principle and step method with a weaker Lipschitz condition. Also, we propose a new algorithm of successive approximation sequence generated by the step method and we give an example to illustrate the applications of the abstract results.

Key Words and Phrases: Integro-differential equation, two time lags, step method, Picard operators, fibre contraction principle.

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