

## FIXED POINT RESULTS FOR WEAK CONTRACTIVE MAPPINGS IN ORDERED $K$ -METRIC SPACES

LJ. ĆIRIĆ\*, B. SAMET\*\*, C. VETRO\*\*\* AND M. ABBAS\*\*\*\*

\* Faculty of Mechanical Engineering, Kraljice Marije 16, 11 000 Belgrade, Serbia  
E-mail: lciric@rcub.bg.ac.rs

\*\* Université de Tunis, Ecole Supérieure des Sciences et Techniques de Tunis, Département de  
Mathématiques, 5, avenue Taha Hussein-Tunis, B.P.:56, Bab Menara-1008, Tunisie  
E-mail: bessem.samet@gmail.com

\*\*\* Department of Mathematics and Informatics, University of Palermo  
Via Archirafi 34, 90123 Palermo, Italy  
E-mail: cvetro@math.unipa.it

\*\*\*\* Centre for Advanced Studies in Mathematics and Department of Mathematics  
Lahore University of Management Sciences, 54792-Lahore, Pakistan  
E-mail: mujahid@lums.edu.pk

**Abstract.** In this paper, we derive new coincidence and common fixed point theorems for self-maps satisfying a weak contractive condition in an ordered  $K$ -metric space. As application, the obtained results are used to prove an existence theorem of solutions of a nonlinear integral equation.

**Key Words and Phrases:** Fixed point, partially ordered set, cone metric space, weak contraction, integral equation.

**2010 Mathematics Subject Classification:** 54H25, 47H10.

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\* First author is supported by Grant No.174025 of the Ministry of Science, Technology and Development, Republic of Serbia.

\*\*\* Third author is supported by Universit degli Studi di Palermo, Local University Project R.S. ex 60 percent.

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*Received: October 4, 2010; Accepted: October 28, 2010.*