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GLOBAL EXISTENCE RESULTS FOR A STOCHASTIC DIFFERENTIAL EQUATION IN HILBERT SPACES

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Abstract. In this paper, we investigate the existence of mild solutions on the whole real axis for a class of stochastic differential equations in a real separable Hilbert space. By using the Banach contraction mapping principle and the fixed point theorem for condensing maps, some global existence results are obtained under some suitable conditions.
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1. Introduction

In this paper we consider the existence of global solutions for a class of stochastic differential equations in the form

$$d[x(t) - g\,(t, x(t))] = Ax(t)dt + G\,(t, x(t))\,dw(t), \;\; t \in \mathbb{R}, \tag{1.1}$$

where A is the infinitesimal generator of an hyperbolic C_0-semigroup of bounded linear operators $\{T(t)\}_{t\geq 0}$ in the Hilbert space $\mathbb{H}$, $g : \mathbb{R} \times \mathbb{H} \to \mathbb{U}$ and $G : \mathbb{R} \times \mathbb{H} \to L_2^0$ are appropriate functions specified later, and $w(t)$ is a Brownian motion.

Stochastic differential equation has attracted great interest due to its applications in characterizing many problems in physics, biology, mechanics and so on. Qualitative properties such as existence, uniqueness and stability for various stochastic differential systems have been extensively studied by many researchers, see for instance [2, 3, 4, 10, 11, 12, 14, 18, 19, 21] and the references therein.

Recently, Hernández in [9] has investigated the existence of global solutions for a class of abstract neutral differential equations of the form

$$\frac{d}{dt}[x(t) + g(t, x_t)] = Ax(t) + f(t, x_t), \;\; t \in \mathbb{R},$$

where A is the infinitesimal generator of an hyperbolic C_0-semigroup of bounded linear operators $\{T(t)\}_{t\geq 0}$ on a Banach space X, the history $x_t : (-\infty, 0] \to X$ defined by $x_t(\theta) = x(t+\theta)$, belongs to some abstract phase space $\mathcal{B}$ defined axiomatically and $g, f : \mathbb{R} \times \mathcal{B} \to X$ are continuous functions. The approach is based upon the Banach contraction principle and Schauder's fixed point theorem, and a fixed point theorem for condensing maps.

Motivated by the above mentioned work [9], the main purpose of this paper is to deal with the existence of global solutions to the problem (1.1). Our results are established by using a fixed point theorem for condensing maps and the Banach contraction mapping principle. The obtained results can be seen as a contribution to this emerging field.

The rest of this paper is organized as follows: In section 2 we recall some basic definitions, lemmas and preliminary facts which will be need in the sequel. Our main results and their proofs are arranged in Section 3.

2. Preliminaries

This section is concerned with some basic concepts, notations, definitions, lemmas and technical results which are used in the sequel. For more details on this section, we refer the reader to [8, 17].

Throughout the paper, $(\mathbb{H}, \|\cdot\|, \langle\cdot,\cdot\rangle)$ and $(\mathbb{K}, \|\cdot\|_{\mathbb{K}}, \langle\cdot,\cdot\rangle_{\mathbb{K}})$ denote two real separable Hilbert spaces. Let $(\Omega, \mathscr{F}, \mathbb{P})$ be a complete probability space. We denote by $L_2(\mathbb{K}, \mathbb{H})$ the set of all Hilbert-Schmidt operators $\Phi : \mathbb{K} \to \mathbb{H}$, equipped with the Hilbert-Schmidt norm $\|\cdot\|_2$.

For a symmetric nonnegative operator $Q \in L_2(\mathbb{K}, \mathbb{H})$ with finite trace we suppose that $\{w(t) : t \in \mathbb{R}\}$ is a Q-Wiener process defined on $(\Omega, \mathscr{F}, \mathbb{P})$ and with values in $\mathbb{K}$. So, actually, w can be obtained as follows: let $w_i(t), t \in \mathbb{R}$, $i = 1, 2$, be independent $\mathbb{K}$-valued Q-Wiener processes, then

$$w(t) = \begin{cases} w_1(t) & \text{if } t \geq 0, \\ w_2(-t) & \text{if } t \leq 0, \end{cases}$$

is a Q-Wiener process with $\mathbb{R}$ as time parameter, $\mathscr{F}_t = \sigma\{w(s) : s \leq t\}$ is the σ-algebra generated by w.

Let $\mathbb{K}_0 = Q^{\frac{1}{2}}\mathbb{K}$ and $L_2^0 = L_2(\mathbb{K}_0, \mathbb{H})$ with respect to the norm

$$\|\Phi\|_{L_2^0}^2 = \|\Phi Q^{\frac{1}{2}}\|_2^2 = Tr(\Phi Q \Phi^*).$$

Let $L^2(\Omega, \mathscr{F}_t, \mathbb{H})$ denote the Hilbert space of all $\mathscr{F}_t$-measurable square integrable random variables with values in $\mathbb{H}$. Let $L^2_{\mathscr{F}_t}(\mathbb{R}, \mathbb{H})$ be the Hilbert space of all square integrable and $\mathscr{F}_t$-adapted processes with values in $\mathbb{H}$.

In what follows, A is the infinitesimal generator of an analytic semigroup of bounded linear operators $\{T(t)\}_{t\geq 0}$ on $\mathbb{H}$ such that $\sigma(A) \cap i\mathbb{R} = \varnothing$. In this case, the set $\sigma_-(A) = \{\lambda \in \sigma(A) : Re(\lambda) < 0\}$ and $\sigma_+(A) = \{\lambda \in \sigma(A) : Re(\lambda) > 0\}$ are closed and disjoint, and there exists $\varpi > 0$ such that

$$\sup\{Re(\lambda) : \lambda \in \sigma_-(A)\} < -\varpi < 0 < \varpi < \inf\{Re(\lambda) : \lambda \in \sigma_+(A)\}.$$

Let $\mathcal{D} \subset \mathbb{R}^2$ be an open bounded set with smooth boundary $\partial\mathcal{D}$ such that $\sigma_+(A) \subset \mathcal{D} \subseteq \mathbb{C}_+ = \{\lambda \in \mathbb{C} : Re(\lambda) > 0\}$ and $P : \mathbb{H} \to \mathbb{H}$ be the operator defined by

$$Px = \frac{1}{2\pi i} \int_{\partial\mathcal{D}} R(\mu; A)x d\mu, \ x \in \mathbb{H},$$

where $\partial\mathcal{D}$ is oriented counterclockwise. In the next result, $\mathbb{H}_1 = P(\mathbb{H})$, $\mathbb{H}_2 = (I - P)(\mathbb{H})$ and $A_1 : \mathbb{H}_1 \to \mathbb{H}$, $A_2 : D(A_2) = \{x \in D(A) : x \in \mathbb{H}_2, Ax \in \mathbb{H}_2\} \to \mathbb{H}_2$, are the operators defined by $A_1 x = Ax$ for $x \in \mathbb{H}_1$ and $A_2 y = Ay$ for $y \in D(A_2)$. The following properties hold by [13] and [9, Proposition 1.1.].

Lemma 2.1. *The following properties are valid:*

(i) The operator P is a projection, $P(\mathbb{H}) \subset D(A^n)$ for all $n \in \mathbb{N}$, $T(t)Px = PT(t)x$ for all $x \in \mathbb{H}$ and $T(t)\mathbb{H}_i \subset \mathbb{H}_i$ for $i = 1, 2$, and every $t \geq 0$.

(ii) $A_1(\mathbb{H}_1) \subset \mathbb{H}_1$, $\sigma(A_1) = \sigma_+(A)$ and $R(\lambda : A_1) = R(\lambda : A)|_{\mathbb{H}_1}$ for all $\lambda \in \rho(A_1)$. Moreover, A_1 is the generator of a C_0-group $\{T_{A_1}(t)\}_{t\geq 0}$ on $\mathbb{H}_1$ and $T_{A_1}(t) = T(t)|_{\mathbb{H}_1}$ for every $t \geq 0$.

(iii) $\sigma(A_2) = \sigma_-(A)$, $R(\lambda : A_2) = R(\lambda : A)|_{\mathbb{H}_2}$ for all $\lambda \in \rho(A_2)$, A_2 is the generator of an uniformly stable C_0-semigroup $\{T_{A_2}(t)\}_{t\geq 0}$ on $\mathbb{H}_2$, $T_{A_2}(t) = T(t)|_{\mathbb{H}_2}$ for every $t \geq 0$ and $T(t) = T_{A_1}(t) + T_{A_2}(t)$ for each $t \geq 0$.

(iv) There are constants d_i, M_i, $i \in \mathbb{N}$, such that $\|A^i T(t)(I - P)\| \leq M_i e^{-\delta t} t^{-i}$ and $\|A^i T(-t)P\| \leq d_i e^{-\delta t}$ for every $t \geq 0$ and each $i \in \mathbb{N}$.

In this paper, $BC^2(\mathbb{R}, \mathbb{H})$ stands for the collection of all $\mathscr{F}_t$-adapted measurable stochastic processes $x : \mathbb{R} \to \mathbb{H}$, which are square integrable and bounded continuous. It is then easy to check that $BC^2(\mathbb{R}, \mathbb{H})$ is a Banach space when it is endowed with the norm:

$$\|x\|_\infty = \left(\sup_{t\in\mathbb{R}} E\|x(t)\|^2 \right)^{\frac{1}{2}}.$$

We let $L(\mathbb{K}, \mathbb{H})$ denote the space of all linear bounded operators from $\mathbb{K}$ into $\mathbb{H}$, equipped with the usual operator norm $\| \cdot \|_{L(\mathbb{K},\mathbb{H})}$. In addition, the notation $BC_0^2(\mathbb{R}, \mathbb{H}) = \{x \in BC^2(\mathbb{R}, \mathbb{H}) : \lim_{t\to\pm\infty} E\|x(t)\|^2 = 0\}$.

To establish our main theorem, we need the following lemma which can be seen as an immediate consequence of [9, Lemma 2.1.].

Lemma 2.2. *A set $\mathbb{B} \subset BC_0^2(\mathbb{R}, \mathbb{H})$ is relatively compact in $BC_0^2(\mathbb{R}, \mathbb{H})$ if, and only if, $\mathbb{B}(t) = \{x(t) : x \in \mathbb{B}\}$ is relatively compact in $\mathbb{H}$ for every $t \in \mathbb{R}$, $\mathbb{B}$ is equicontinuous and $\lim_{t\to\pm\infty} E\|x(t)\|^2 = 0$ uniformly for $x \in \mathbb{B}$.*

Some of our results are based upon the following fixed point theorem [15, 20].

Lemma 2.3. *Let $\mathfrak{D}$ be a convex, bounded and closed subset of a Banach space $\mathbb{X}$ and $\Lambda : \mathfrak{D} \to \mathfrak{D}$ be a condensing map. Then Λ has a fixed point in $\mathfrak{D}$.*

From Lunardi [13] and Hernández [5, 8, 9], we adopt the following concept of mild solution for the problem (1.1).

Definition 2.1. *An $\mathscr{F}_t$-adapted stochastic process $x(t) \in BC^2(\mathbb{R}, \mathbb{H})$ is called a mild solution of the problem (1.1) if*

$$\begin{aligned} x(t) &= g(t, x(t)) + \int_{-\infty}^{t} AT(t-s)(I-P)g(s, x(s))\, ds \\ &\quad - \int_{t}^{\infty} AT(t-s)Pg(s, x(s))\, ds \\ &\quad + \int_{-\infty}^{t} T(t-s)(I-P)G(s, x(s))\, dw(s) \\ &\quad - \int_{t}^{\infty} T(t-s)PG(s, x(s))\, dw(s), \ t \in \mathbb{R}. \end{aligned}$$

Now we list the following basic assumptions of this paper:
(H1) Let $(\mathbb{U}, \|\cdot\|_{\mathbb{U}}, \langle\cdot,\cdot\rangle_{\mathbb{U}})$ denote an arbitrary real separable Hilbert space. Suppose that $\mathbb{U}$ continuously included in $\mathbb{H}$ and there are functions $H, \tilde{H} \in L^1_{loc}([0,\infty), (0,\infty))$ and $\delta > 0$ with $e^{-\delta s}H(s) \in L^1([0,\infty))$ such that $\|AT(t)\|_{L(\mathbb{U},\mathbb{H})} \leq \tilde{H}(t)$ and $\|AT(t)(I-P)\|_{L(\mathbb{U},\mathbb{H})} \leq e^{-\delta t}H(t)$ for every $t \geq 0$.
(H2) The function $g : \mathbb{R} \times \mathbb{H} \to \mathbb{U}$ is continuous and there exists a constant $L_g > 0$ such that the function satisfies the Lipschitz condition

$$E\|g(t,x) - g(t,y)\|^2_{\mathbb{U}} \leq L_g E\|x-y\|^2,$$

for all $t \in \mathbb{R}$ and for each $x, y \in \mathbb{H}$. Moreover, there exists a constant $M_g > 0$ such that $E\|g(t,0)\|^2_{\mathbb{U}} \leq M_g$ for all $t \in \mathbb{R}$.
(H3) The function $G : \mathbb{R} \times \mathbb{H} \to L_2^0$ is continuous and there exists a constant $L_G > 0$ such that the function satisfies the Lipschitz condition

$$E\|G(t,x) - G(t,y)\|^2_{L_2^0} \leq L_G E\|x-y\|^2,$$

for all $t \in \mathbb{R}$, and for each $x, y \in \mathbb{H}$. Moreover, there exists a constant $M_G > 0$ such that $E\|G(t,0)\|^2_{L_2^0} \leq M_G$ for all $t \in \mathbb{R}$.
(H4) We denote by i_c the inclusion map from $\mathbb{U}$ into $\mathbb{H}$.
(H5) The semigroup $\{T(t)\}_{t>0}$ is compact.
(H6) The function $G : \mathbb{R} \times \mathbb{H} \to L_2^0$ is continuous and there exist an integrable function $m_G : [0,\infty) \to (0,\infty)$ and a continuous non-decreasing function $W : [0,\infty) \to [0,\infty)$ such that

$$E\|G(t,x)\|^2_{L_2^0} \leq m_G(t) W(E\|x\|^2),$$

for every $(t,x) \in \mathbb{R} \times \mathbb{H}$.

Remark 2.1. *Note that the assumption (H1) is achieved in many cases, see, for instance, Lunardi* [13], *and we refer the reader to* [1, 5, 6, 7] *for additional details related this type of condition in the theory of neutral equations.*

In order to proof our main result Theorem 3, we give a useful lemma appeared in [9].

Lemma 2.4. *If the semigroup $\{T(t)\}_{t>0}$ is compact, then $i_c : \mathbb{U} \to \mathbb{H}$ is compact.*

3. Main results

In this section, we present and prove our main results. Firstly, we give an existence and uniqueness result for the problem (1.1).

Theorem 3.1. Assume the conditions (H1)-(H4) are satisfied, then the problem (1.1) has a unique mild solution on $\mathbb{R}$ provide that

$$\begin{aligned} L_0 &= 5\left\{L_g\left[\|i_c\|^2_{L(\mathbb{U},\mathbb{H})}+\left(\int_0^\infty e^{-\delta s}H(s)ds\right)^2+\frac{d_1^2\|i_c\|^2_{L(\mathbb{U},\mathbb{H})}}{\delta^2}\right]\right. \\ &\left.+TrQL_G\left(\frac{M_0^2}{2\delta}+\frac{d_0^2}{2\delta}\right)\right\}<1. \end{aligned} \tag{3.1}$$

Proof. Let $\Gamma : BC^2(\mathbb{R},\mathbb{H}) \to BC^2(\mathbb{R},\mathbb{H})$ be the operator defined by

$$\begin{aligned} \Gamma x(t) &= g(t,x(t))+\int_{-\infty}^t AT(t-s)(I-P)g(s,x(s))\,ds \\ &- \int_t^\infty AT(t-s)Pg(s,x(s))\,ds \\ &+ \int_{-\infty}^t T(t-s)(I-P)G(s,x(s))\,dw(s) \\ &- \int_t^\infty T(t-s)PG(s,x(s))\,dw(s),\ t\in\mathbb{R}. \end{aligned}$$

First we prove that Γx is well defined. From Lemma 2.1 and the estimate

$$\begin{aligned} & E\left\|\int_{-\infty}^t AT(t-s)(I-P)g(s,x(s))\,ds\right\|^2 \\ \le\ & E\left(\int_{-\infty}^t e^{-\delta(t-s)}H(t-s)[\|g(s,x(s))-g(s,0)\|_U+\|g(s,0)\|_U]ds\right)^2 \\ \le\ & \left(\int_{-\infty}^t e^{-\delta(t-s)}H(t-s)ds\right) \\ & \times\left(\int_{-\infty}^t e^{-\delta(t-s)}H(t-s)E[\|g(s,x(s))-g(s,0)\|_U+\|g(s,0)\|_U]^2ds\right) \\ \le\ & \left(\int_{-\infty}^t e^{-\delta(t-s)}H(t-s)ds\right) \\ & \times\left(\int_{-\infty}^t e^{-\delta(t-s)}H(t-s)[2L_gE\|x(s)\|^2+2E\|g(s,0)\|_U^2]ds\right) \\ \le\ & \left[2L_g\sup_{t\in\mathbb{R}}E\|x(t)\|^2+2\sup_{t\in\mathbb{R}}E\|g(t,0)\|_U^2\right]\left(\int_0^\infty e^{-\delta s}H(s)ds\right)^2 \\ =\ & \left(2L_g\|x\|_\infty^2+2M_g\right)\left(\int_0^\infty e^{-\delta s}H(s)ds\right)^2, \end{aligned}$$

we infer that the function $s \to AT(t-s)(I-P)g(s,x(s))$ is integrable on $(-\infty,t)$ for every $t \in \mathbb{R}$ and the function $s \to \int_{-\infty}^{t} AT(t-s)(I-P)g(s,x(s))\,ds$ belongs to $BC^2(\mathbb{R},\mathbb{H})$.

Similarly, from the estimate

$$
\begin{aligned}
& E\left\|\int_t^{\infty} AT(t-s)Pg(s,x(s))\,ds\right\|^2 \\
\leq\ & d_1^2 E\left(\int_t^{\infty} e^{\delta(t-s)}[\|i_c\|_{L(\mathbb{U},\mathbb{H})}\|g(s,x(s))-g(s,0)\|_U + \|i_c\|_{L(\mathbb{U},\mathbb{H})}\|g(s,0)\|_U]ds\right)^2 \\
\leq\ & d_1^2\left(\int_t^{\infty} e^{\delta(t-s)}ds\right) \\
& \times\left(\int_t^{\infty} e^{\delta(t-s)}E[\|i_c\|_{L(\mathbb{U},\mathbb{H})}\|g(s,x(s))-g(s,0)\|_U + \|i_c\|_{L(\mathbb{U},\mathbb{H})}\|g(s,0)\|_U]^2ds\right) \\
\leq\ & d_1^2\left(\int_t^{\infty} e^{\delta(t-s)}ds\right) \\
& \times\left(\int_t^{\infty} e^{\delta(t-s)}[2\|i_c\|^2_{L(\mathbb{U},\mathbb{H})}L_g E\|x(s)\|^2 + 2\|i_c\|^2_{L(\mathbb{U},\mathbb{H})}E\|g(s,0)\|_U^2]ds\right) \\
\leq\ & 2d_1^2\|i_c\|^2_{L(\mathbb{U},\mathbb{H})}\left(L_g\|x\|_\infty^2 + M_g\right)\left(\int_t^{\infty} e^{\delta(t-s)}ds\right)^2 \\
\leq\ & \frac{d_1^2}{\delta^2}2\|i_c\|^2_{L(\mathbb{U},\mathbb{H})}\left(L_g\|x\|_\infty^2 + M_g\right),
\end{aligned}
$$

it follows that $s \to AT(t-s)Pg(s,x(s))$ is integrable on (t,∞) for all $t \in \mathbb{R}$ and that $s \to \int_t^{\infty} AT(t-s)Pg(s,x(s))\,ds \in BC^2(\mathbb{R},\mathbb{H})$. Arguing as above, we can complete the proof that $\Gamma x \in BC^2(\mathbb{R},\mathbb{H})$. Therefore, Γ is well defined on $BC^2(\mathbb{R},\mathbb{H})$.

Now the remaining task is to prove that Γ is a strict contraction on $BC^2(\mathbb{R},\mathbb{H})$. Indeed, for each $t \in \mathbb{R}$, $x, y \in BC^2(\mathbb{R},\mathbb{H})$, we see that

$$
\begin{aligned}
& E\|\Gamma x(t) - \Gamma y(t)\|^2 \\
\leq\ & 5E\|g(t,x(t)) - g(t,y(t))\|_U^2 \\
& +5E\left(\left\|\int_{-\infty}^{t} AT(t-s)(I-P)[g(s,x(s)) - g(s,y(s))]ds\right\|\right)^2 \\
& +5E\left(\left\|\int_t^{\infty} AT(t-s)P[g(s,x(s)) - g(s,y(s))]ds\right\|\right)^2 \\
& +5E\left(\left\|\int_{-\infty}^{t} T(t-s)(I-P)[G(s,x(s)) - G(s,y(s))]dw(s)\right\|\right)^2 \\
& +5E\left(\left\|\int_t^{\infty} T(t-s)P[G(s,x(s)) - G(s,y(s))]dw(s)\right\|\right)^2 \\
\leq\ & 5\|i_c\|^2_{L(\mathbb{U},\mathbb{H})}E\|g(t,x(t)) - g(t,y(t))\|_U^2
\end{aligned}
$$

$$
\begin{aligned}
&+5E\left(\int_{-\infty}^{t} e^{-\delta(t-s)}H(t-s)\|g\left(s,x(s)\right)-g\left(s,y(s)\right)\|_U ds\right)^2\\
&+5d_1^2E\left(\int_{t}^{\infty} e^{\delta(t-s)}\|i_c\|_{L(\mathbb{U},\mathbb{H})}\|g\left(s,x(s)\right)-g\left(s,y(s)\right)\|_U ds\right)^2\\
&+5TrQE\left(\int_{-\infty}^{t}\|T(t-s)(I-P)[G\left(s,x(s)\right)-G\left(s,y(s)\right)]\|^2 ds\right)\\
&+5TrQE\left(\int_{t}^{\infty}\|T(t-s)P[G\left(s,x(s)\right)-G\left(s,y(s)\right)]\|^2 ds\right)\\
\leq\;& 5\|i_c\|_{L(\mathbb{U},\mathbb{H})}^2L_g\sup_{t\in\mathbb{R}}E\|x(t)-y(t)\|^2\\
&+5L_g\left(\int_{-\infty}^{t} e^{-\delta(t-s)}H(t-s)ds\right)^2\sup_{t\in\mathbb{R}}E\|x(t)-y(t)\|^2\\
&+5d_1^2\|i_c\|_{L(\mathbb{U},\mathbb{H})}^2L_g\left(\int_{t}^{\infty} e^{\delta(t-s)}ds\right)^2\sup_{t\in\mathbb{R}}E\|x(t)-y(t)\|^2\\
&+5TrQM_0^2L_G\left(\int_{-\infty}^{t} e^{-2\delta(t-s)}ds\right)\sup_{t\in\mathbb{R}}E\|x(t)-y(t)\|^2\\
&+5TrQd_0^2L_G\left(\int_{t}^{\infty} e^{2\delta(t-s)}ds\right)\sup_{t\in\mathbb{R}}E\|x(t)-y(t)\|^2\\
\leq\;& 5\|i_c\|_{L(\mathbb{U},\mathbb{H})}^2L_g\|x-y\|_\infty^2+5L_g\left(\int_{-\infty}^{t} e^{-\delta(t-s)}H(t-s)ds\right)^2\|x-y\|_\infty^2\\
&+5L_g\frac{d_1^2\|i_c\|_{L(\mathbb{U},\mathbb{H})}^2}{\delta^2}\|x-y\|_\infty^2+5TrQL_G\frac{M_0^2}{2\delta}\|x-y\|_\infty^2+5TrQL_G\frac{d_0^2}{2\delta}\|x-y\|_\infty^2\\
=\;& \Bigg\{5L_g\left[\|i_c\|_{L(\mathbb{U},\mathbb{H})}^2+\left(\int_{0}^{\infty} e^{-\delta s}H(s)ds\right)^2+\frac{d_1^2\|i_c\|_{L(\mathbb{U},\mathbb{H})}^2}{\delta^2}\right]\\
&+5TrQL_G\left(\frac{M_0^2}{2\delta}+\frac{d_0^2}{2\delta}\right)\Bigg\}\|x-y\|_\infty^2\\
=\;& L_0\|x-y\|_\infty^2.
\end{aligned}
$$

Hence, we obtain

$$\|\Gamma x-\Gamma y\|_\infty^2\leq L_0\|x-y\|_\infty^2,$$

which implies that Γ is a contraction by (3.1). So by the Banach contraction principle, we conclude that there exists a unique fixed point $x(\cdot)$ for Γ in $BC^2\left(\mathbb{R},\mathbb{H}\right)$, therefore the problem (1.1) has a unique mild solution on $\mathbb{R}$. The proof is completed.

Next, we establish an existence result of mild solutions to the problem (1.1) via fixed point theorem for condensing maps.

Theorem 3.2. Assume the conditions (H1), (H2) and (H4)-(H6) hold, then the problem (1.1) admits at least one mild solution on $\mathbb{R}$ provide that

$$L_1 = \sup_{t\in\mathbb{R}} \left[M_0^2 \int_{-\infty}^{t} e^{-2\delta(t-s)} m_G(s)ds + d_0^2 \int_t^{\infty} e^{2\delta(t-s)} m_G(s)ds \right] < \infty \tag{3.2}$$

and

$$\begin{aligned} 10\|i_c\|^2_{L(\mathbb{U},\mathbb{H})} L_g + 10L_g \left(\int_0^{\infty} e^{-\delta s} H(s)ds \right)^2 \\ +10L_g\|i_c\|^2_{L(\mathbb{U},\mathbb{H})} \frac{d_1^2}{\delta^2} + 5TrQL_1 \liminf_{r\to\infty} \frac{W(r)}{r} < 1. \end{aligned} \tag{3.3}$$

Proof. We define the operator $\Gamma : BC^2(\mathbb{R},\mathbb{H}) \to BC^2(\mathbb{R},\mathbb{H})$ as

$$\begin{aligned} \Gamma x(t) &= g(t,x(t)) + \int_{-\infty}^{t} AT(t-s)(I-P)g(s,x(s))\,ds \\ &\quad - \int_t^{\infty} AT(t-s)Pg(s,x(s))\,ds \\ &\quad + \int_{-\infty}^{t} T(t-s)(I-P)G(s,x(s))\,dw(s) \\ &\quad - \int_t^{\infty} T(t-s)PG(s,x(s))\,dw(s), \end{aligned}$$

$t \in \mathbb{R}$. From Theorem 3 and the assumptions (H5)-(H6), we infer that Γ is well defined on $BC^2(\mathbb{R},\mathbb{H})$. Our proof will be given in several steps.

Step 1. Let $B_r = \{x \in BC^2(\mathbb{R},\mathbb{H}) : E\|x\|^2 \le r\}$ for each $r > 0$. Clearly, for each positive number r, B_r is a bounded closed convex set in $BC^2(\mathbb{R},\mathbb{H})$. We claim that there exists a positive number r such that $\Gamma(B_r) \subset B_r$. If it is not true, then for each positive number r, there would exist $x_r \in B_r$ and $t_r \in \mathbb{R}$ such that $E\|\Gamma x_r(t_r)\|^2 > r$. However, on the other hand, we have

$$\begin{aligned} & r < E\|\Gamma x_r(t_r)\|^2 \\ \le\ & 5E\|g(t_r,x_r(t_r))\|^2_U + 5E\left(\left\| \int_{-\infty}^{t_r} AT(t_r-s)(I-P)g(s,x_r(s))\,ds \right\| \right)^2 \\ & +5E\left(\left\| \int_{t_r}^{\infty} AT(t_r-s)Pg(s,x_r(s))\,ds \right\| \right)^2 \\ & +5E\left(\left\| \int_{-\infty}^{t_r} T(t_r-s)(I-P)G(s,x_r(s))\,dw(s) \right\| \right)^2 \\ & +5E\left(\left\| \int_{t_r}^{\infty} T(t_r-s)PG(s,x_r(s))\,dw(s) \right\| \right)^2 \\ \le\ & 10\|i_c\|^2_{L(\mathbb{U},\mathbb{H})} L_g E\|x_r(t_r)\|^2 + 10\|i_c\|^2_{L(\mathbb{U},\mathbb{H})} E\|g(t_r,0)\|^2_U \\ & + \left[10L_g \sup_{t\in\mathbb{R}} E\|x_r(t)\|^2 + 10\sup_{t\in\mathbb{R}} E\|g(t,0)\|^2_U \right] \left(\int_0^{\infty} e^{-\delta s} H(s)ds \right)^2 \end{aligned}$$

$$
\begin{aligned}
&+10\|i_c\|^2_{L(\mathbb{U},\mathbb{H})}\frac{d_1^2}{\delta^2}\left(L_g\sup_{t\in\mathbb{R}}E\|x_r(t)\|^2+\sup_{t\in\mathbb{R}}E\|g\left(t,0\right)\|^2_U\right)\\
&+5TrQM_0^2\int_{-\infty}^{t_r}e^{-2\delta(t_r-s)}m_G(s)W(E\|x_r(s)\|^2)ds\\
&+5TrQd_0^2\int_{t_r}^{\infty}e^{2\delta(t_r-s)}m_G(s)W(E\|x_r(s)\|^2)ds\\
\leq\ &10\|i_c\|^2_{L(\mathbb{U},\mathbb{H})}L_gr+10\|i_c\|^2_{L(\mathbb{U},\mathbb{H})}M_g+[10L_gr+10M_g]\left(\int_0^{\infty}e^{-\delta s}H(s)ds\right)^2\\
&+10\|i_c\|^2_{L(\mathbb{U},\mathbb{H})}\frac{d_1^2}{\delta^2}(L_gr+M_g)\\
&+5TrQW(r)\left(M_0^2\int_{-\infty}^{t_r}e^{-2\delta(t_r-s)}m_G(s)ds+d_0^2\int_{t_r}^{\infty}e^{2\delta(t_r-s)}m_G(s)ds\right)\\
\leq\ &10\|i_c\|^2_{L(\mathbb{U},\mathbb{H})}L_gr+10\|i_c\|^2_{L(\mathbb{U},\mathbb{H})}M_g+[10L_gr+10M_g]\left(\int_0^{\infty}e^{-\delta s}H(s)ds\right)^2\\
&+10\|i_c\|^2_{L(\mathbb{U},\mathbb{H})}\frac{d_1^2}{\delta^2}(L_gr+M_g)+5TrQW(r)L_1.
\end{aligned}
$$

Dividing both sides by r and taking the lower limit as $r\to\infty$, we obtain

$$
\begin{aligned}
1\ \leq\ &10\|i_c\|^2_{L(\mathbb{U},\mathbb{H})}L_g+10L_g\left(\int_0^{\infty}e^{-\delta s}H(s)ds\right)^2+10L_g\|i_c\|^2_{L(\mathbb{U},\mathbb{H})}\frac{d_1^2}{\delta^2}\\
&+5TrQL_1\liminf_{r\to\infty}\frac{W(r)}{r},
\end{aligned}
$$

which contradicts the condition (3.3). Thus, for some positive number r, $\Gamma(B_r)\subset B_r$. In what follows, we aim to show that the operator Γ is condensing on B_r. Now we decompose Γ as $\Gamma=\Gamma_1+\Gamma_2+\Gamma_3$, where the operators Γ_1, Γ_2, Γ_3 are defined on B_r, respectively, by

$$
\begin{aligned}
\Gamma_1x(t)\ =\ &g\left(t,x(t)\right)+\int_{-\infty}^{t}AT(t-s)(I-P)g\left(s,x(s)\right)ds\\
&-\int_t^{\infty}AT(t-s)Pg\left(s,x(s)\right)ds,
\end{aligned}
$$

$$\Gamma_2x(t)=\int_{-\infty}^{t}T(t-s)(I-P)G\left(s,x(s)\right)dw(s),$$

$$\Gamma_3x(t)=-\int_t^{\infty}T(t-s)PG\left(s,x(s)\right)dw(s),\ \ t\in\mathbb{R}.$$

We will verify that Γ_1 is a contraction while Γ_2 and Γ_3 are completely continuous.

Step 2. Γ_1 is a contraction. Let $x,y\in B_r$. Then for each $t\in\mathbb{R}$ and by condition (H2), we have

$$
\begin{aligned}
&E\|\Gamma_1x(t)-\Gamma_1y(t)\|^2\\
\leq\ &3E\|g\left(t,x(t)\right)-g\left(t,y(t)\right)\|^2_U
\end{aligned}
$$

$$
\begin{aligned}
&+3E\left(\left\|\int_{-\infty}^{t} AT(t-s)(I-P)[g(s,x(s))-g(s,y(s))]ds\right\|\right)^2 \\
&+3E\left(\left\|\int_{t}^{\infty} AT(t-s)P[g(s,x(s))-g(s,y(s))]ds\right\|\right)^2 \\
\leq\ & 3\|i_c\|^2_{L(\mathbb{U},\mathbb{H})} E\|g(t,x(t))-g(t,y(t))\|^2_U \\
&+3\left(\int_{-\infty}^{t} e^{-\delta(t-s)}H(t-s)ds\right) \\
&\times\left(\int_{-\infty}^{t} e^{-\delta(t-s)}H(t-s)E\|g(s,x(s))-g(s,y(s))\|^2_U ds\right) \\
&+3d_1^2\left(\int_{t}^{\infty} e^{\delta(t-s)}ds\right) \\
&\times\left(\int_{t}^{\infty} e^{\delta(t-s)}\|i_c\|^2_{L(\mathbb{U},\mathbb{H})} E\|g(s,x(s))-g(s,y(s))\|^2_U ds\right) \\
\leq\ & 3\|i_c\|^2_{L(\mathbb{U},\mathbb{H})} L_g \sup_{t\in\mathbb{R}} E\|x(t)-y(t)\|^2 \\
&+3L_g\left(\int_{-\infty}^{t} e^{-\delta(t-s)}H(t-s)ds\right)^2 \sup_{t\in\mathbb{R}} E\|x(t)-y(t)\|^2 \\
&+3d_1^2\|i_c\|^2_{L(\mathbb{U},\mathbb{H})} L_g\left(\int_{t}^{\infty} e^{\delta(t-s)}ds\right)^2 \sup_{t\in\mathbb{R}} E\|x(t)-y(t)\|^2 \\
\leq\ & 3L_g\left[\|i_c\|^2_{L(\mathbb{U},\mathbb{H})} + \left(\int_0^{\infty} e^{-\delta s}H(s)ds\right)^2 + \frac{d_1^2\|i_c\|^2_{L(\mathbb{U},\mathbb{H})}}{\delta^2}\right]\|x-y\|^2_\infty \\
=\ & L_2\|x-y\|^2_\infty,
\end{aligned}
$$

where $L_2 = 3L_g\left[\|i_c\|^2_{L(\mathbb{U},\mathbb{H})} + \left(\int_0^{\infty} e^{-\delta s}H(s)ds\right)^2 + \frac{d_1^2\|i_c\|^2_{L(\mathbb{U},\mathbb{H})}}{\delta^2}\right]$.

Thus

$$\|\Gamma_1 x - \Gamma_1 y\|^2_\infty \leq L_2\|x-y\|^2_\infty,$$

which implies that Γ_1 is a contraction by (3.3).

Step 3. Γ_2 is completely continuous.

(a) For all $t\in\mathbb{R}$, the set $\Gamma_2 B_r(t) = \{\Gamma_2 x(t) : x\in B_r\}$ is relatively compact in $\mathbb{H}$. In fact, for each $t\in\mathbb{R}$, $x\in B_r$ and for any $\varepsilon>0$, we see that

$$
\begin{aligned}
\Gamma_2 x(t) &= T(\varepsilon)\int_{-\infty}^{t-\varepsilon} T(t-\varepsilon-s)(I-P)G(s,x(s))\,dw(s) \\
&\quad + \int_{t-\varepsilon}^{t} T(t-s)(I-P)G(s,x(s))\,dw(s).
\end{aligned}
$$

Moreover, from the estimate

$$E\left\|\int_{-\infty}^{t-\varepsilon} T(t-\varepsilon-s)(I-P)G(s,x(s))\,dw(s)\right\|^2$$

$$
\begin{aligned}
&\leq \; TrQW(r)\int_{-\infty}^{t-\varepsilon} M_0^2 e^{-2\delta(t-\varepsilon-s)} m_G(s)ds \\
&\leq \; TrQW(r)L_1
\end{aligned}
$$

and

$$E\left\|\int_{t-\varepsilon}^{t} T(t-s)(I-P)G\left(s,x(s)\right)dw(s)\right\|^2 \leq TrQW(r)M_0^2\int_{t-\varepsilon}^{t} m_G(s)ds,$$

we obtain that

$$\Gamma_2 B_r(t) \subset T(\varepsilon)B_{r^*}(t) + C_\varepsilon, \tag{3.4}$$

where $r^* = TrQW(r)L_1$ and $diam(C_\varepsilon) \leq TrQW(r)M_0^2\int_{t-\varepsilon}^{t} m_G(s)ds$. Since $T(\varepsilon)$ is compact, $diam(C_\varepsilon) \to 0$ as $\varepsilon \to 0$ and $x(\cdot)$ is arbitrary, from (3.4) we infer that is relatively compact in $\mathbb{H}$.

(b) The set $\Gamma_2 B_r = \{\Gamma_2 x : x \in B_r\}$ is equicontinuous.

Let ε be small enough and $t \in \mathbb{R}$. Since $\Psi = \Gamma_2 B_r(t)$ is relatively compact in $\mathbb{H}$, there exists $\gamma > 0$ such that $E\|\left(T(h)-I\right)y\|^2 \leq \varepsilon$ and $\int_t^{t+h} m_G(s)ds \leq \varepsilon$ for all $y \in \Psi$ and every $0 < h < \gamma$. Then, for $x \in B_r$ and $0 < h < \gamma$ we have

$$
\begin{aligned}
&\quad\; E\|\Gamma_2 u(t+h) - \Gamma_2 u(t)\|^2 \\
&\leq \; 2E\left\|(T(h)-I)\int_{-\infty}^{t} T(t-s)(I-P)G\left(s,x(s)\right)dw(s)\right\|^2 \\
&\quad\; +2E\left\|\int_{t}^{t+h} T(t+h-s)(I-P)G\left(s,x(s)\right)dw(s)\right\|^2 \\
&\leq \; 2\sup_{y\in\Psi} E\|\left(T(h)-I\right)y\|^2 + 2TrQW(r)M_0^2\int_t^{t+h} m_G(s)ds \\
&\leq \; 2\varepsilon\left(1 + TrQW(r)M_0^2\right),
\end{aligned}
$$

which implies that the set $\Gamma_2 B_r$ is right equicontinuous at t. By a similar procedure we can show that $\Gamma_2 B_r$ is left equicontinuous at t. Thus, the set $\Gamma_2 B_r$ is equicontinuous.

(c) $\lim_{t\to\pm\infty} E\|\Gamma_2 x(t)\|^2 = 0$ uniformly for $x \in B_r$.

Let $\varepsilon > 0$ be given, we select $N_\varepsilon \in \mathbb{N}$ such that

$$TrQM_0^2\int_{-\infty}^{t} e^{-2\delta(t-s)} m_G(s)ds < \varepsilon,\; t \leq -N_\varepsilon,$$

$$2M_0^2 e^{-2\delta N_\varepsilon} r + 2TrQM_0^2 W(r)\sup_{\vartheta\geq N_\varepsilon}\int_{N_\varepsilon}^{\vartheta} e^{-2\delta(\vartheta-s)} m_G(s)ds < \varepsilon,\; t \geq 2N_\varepsilon.$$

Consequently, for $x \in B_r$ and $t \leq -N_\varepsilon$, we find that

$$
\begin{aligned}
E\|\Gamma_2 x(t)\|^2 \;&\leq\; TrQ\int_{-\infty}^{t} \|T(t-s)(I-P)\|^2 E\|G\left(s,x(s)\right)\|^2_{L_2^0} \\
&\leq\; TrQM_0^2\int_{-\infty}^{t} e^{-2\delta(t-s)} m_G(s)W(r)ds \leq \varepsilon W(r),
\end{aligned}
$$

which shows that $\lim_{t\to-\infty} E\|\Gamma_2 x(t)\|^2 = 0$, uniformly for $x \in B_r$. On the other hand, for $t \geq 2N_\varepsilon$ and $x \in B_r$, we get

$$\begin{aligned}
E\|\Gamma_2 x(t)\|^2 &\leq 2E\left(\left\|T(t-N_\varepsilon)\int_{-\infty}^{N_\varepsilon} T(N_\varepsilon - s)(I-P)G\left(s,x(s)\right)dw(s)\right\|\right)^2 \\
&\quad +2E\left\|\int_{N_\varepsilon}^{t} T(t-s)(I-P)G\left(s,x(s)\right)dw(s)\right\|^2 \\
&\leq 2M_0^2 e^{-2\delta(t-N_\varepsilon)}E\|\Gamma_2 x(N_\varepsilon)\|^2 \\
&\quad +2TrQM_0^2 W(r)\int_{N_\varepsilon}^{t} e^{-2\delta(t-s)}m_G(s)ds \\
&\leq 2M_0^2 e^{-2\delta N_\varepsilon} r + 2TrQM_0^2 W(r)\sup_{\vartheta\geq N_\varepsilon}\int_{N_\varepsilon}^{\vartheta} e^{-2\delta(\vartheta-s)}m_G(s)ds \\
&\leq \varepsilon,
\end{aligned}$$

which implies that $\lim_{t\to\infty} E\|\Gamma_2 x(t)\|^2 = 0$ uniformly for $x \in B_r$.

As a consequence of the above steps and Lemma 2.2, we can conclude that Γ_2 is completely continuous on B_r. Moreover, applying the same method as in Step 3 of this proof, we obtain that Γ_3 is also completely continuous on B_r. These arguments enable us to conclude that $\Gamma = \Gamma_1 + \Gamma_2 + \Gamma_3$ is a condensing map on B_r.

Now, from Lemma 2.3, we assert that the problem (1.1) has a mild solution on $\mathbb{R}$. The proof is now completed.

4. Applications

In this section we consider a simple example of our abstract results. We examine the existence and uniqueness of global mild solutions to the partial neutral stochastic differential system

$$d\left[x(t,\xi) - \int_0^\pi b(\eta,\xi)x(t,\eta)d\eta\right] = \frac{\partial^2}{\partial\xi^2}x(t,\xi)dt + a\left(t,x(t,\xi)\right)dw(t), \tag{4.1}$$

$$x(t,0) = x(t,\pi) = 0, \tag{4.2}$$

for all $(t,\xi) \in \mathbb{R}\times[0,\pi]$, where $w(t)$ is a Brownian motion.

Let $H := L^2([0,\pi])$ with the norm $\|\cdot\|$ and A be the operator defined by $Az = z''$, with domain

$$D(A) = \{z \in H : z'' \in H, z(0) = z(\pi) = 0\}.$$

It is well known that (for example, see [9, 13, 16]) A is the infinitesimal generator of an analytic semigroup $\{T(t)\}_{t\geq 0}$ on H. Furthermore, A has a discrete spectrum with eigenvalues of the form $-n^2$, $n \in \mathbb{N}$, and corresponding normalized eigenvectors given by $z_n(\xi) = \sqrt{\frac{2}{\pi}}\sin(n\xi)$. Moreover, the following properties hold:

(1) $\{z_n : n \in \mathbb{N}\}$ is an orthonormal basis of H.
(2) $T(t)z = \sum_{n=1}^{\infty} e^{-n^2 t}\langle z, z_n\rangle z_n$, for every $z \in H$ and all $t > 0$.
(3) $Az = -\sum_{n=1}^{\infty} n^2\langle z, z_n\rangle z_n$, for every $z \in D(A)$.
(4) $\|T(t)\| \leq e^{-t}$ for every $t \geq 0$.

In addition, it is possible to define the fractional power $(-A)^\alpha$, for $0 < \alpha \leq 1$, as closed linear operator on its domain $D((-A)^\alpha)$ with inverse $(-A)^{-\alpha}$ (see [16] and [13] for details). Especially,

(5) For $z \in H$ and $\alpha \in (0,1)$, $(-A)^{-\alpha} z = \sum_{n=1}^{\infty} \frac{1}{n^{2\alpha}} \langle z, z_n \rangle z_n$.

(6) The operator $(-A)^\alpha : D((-A)^\alpha) \subseteq H \to H$ is given by $(-A)^\alpha z = \sum_{n=1}^{\infty} n^{2\alpha} \langle z, z_n \rangle z_n$ for every $z \in D((-A)^\alpha) = \{z \in H : \sum_{n=1}^{\infty} n^{2\alpha} \langle z, z_n \rangle z_n \in H\}$.

(7) For $\alpha = \frac{1}{2}$, $\|(-A)^{-\frac{1}{2}}\| = 1$ and $\|(-A)^{\frac{1}{2}} T(t)\| \leq \frac{1}{\sqrt{2}} e^{-\frac{t}{2}} t^{-\frac{1}{2}}$ for all $t > 0$.

Now, we take $K = \mathbb{R}$ with the norm $|\cdot|$, and we assume that the following conditions hold:

(H7) The functions $b(\cdot)$, $\frac{\partial^i}{\partial \xi^i} b(\eta, \xi)$, $i = 0, 1$ are Lebesgue measurable, $b(\eta, 0) = b(\eta, \pi) = 0$, and let

$$L_g = \max\left\{ \int_0^\pi \int_0^\pi \left(\frac{\partial^i}{\partial \xi^i} b(\eta, \xi) \right)^2 d\eta d\xi : i = 0, 1 \right\} < \infty.$$

(H8) Let $g : \mathbb{R} \times H \to H_{\frac{1}{2}}$ and $G : \mathbb{R} \times H \to L_2^0$ be defined for $\xi \in [0, \pi]$ and $t \in \mathbb{R}$ by

$$g(t, x)(\xi) = \int_0^\pi b(\eta, \xi) x(\eta) d\eta,$$

$$G(t, x)(\xi) = a(t, x(\xi)).$$

Then the system (4.1)-(4.2) takes the abstract form

$$d[x(t) - g(t, x(t))] = Ax(t)dt + G(t, x(t)) dw(t), \ t \in \mathbb{R}.$$

Moreover, from (H7), it follows that g is continuous and $g(t, \cdot)$ is a bounded linear operator with $\|g(t, \cdot)\|_{L(H, H_{\frac{1}{2}})} \leq L_g$. Further, we can impose some suitable conditions on the above defined function $G(\cdot)$ to verify the assumption on Theorem 3.1, we can conclude that the problem (4.1)-(4.2) has a unique global mild solution.

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