

## AN INVERSE SOURCE PROBLEM IN NEAR ZERO FREQUENCY SOUNDING OF LAYERED MEDIA

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**Abstract.** Geophysicists have long known that near-zero frequency data is insensitive to the variations of the index of refraction in a medium. Using the contraction mapping principle we are able to prove this fact for layered media. Moreover, the method of proof extracts from the data the part which is sensitive to the variation of the refractive index. We illustrate this sensitivity by reconstructing an approximate refractive index. The approximation is necessary due to the ill-posedness in the reconstruction problem.

**Key Words and Phrases:** Frequency sounding, contraction mapping principle, asymptotic expansion, inverse Riccati equations.

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