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COINCIDENCE POINTS OF MULTIVALUED $f-$ALMOST NONEXPANSIVE MAPPINGS

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Abstract. V. Berinde and M. Păcurar [V. Berinde and M. Păcurar, Fixed points and continuity of almost contractions, Fixed Point Theory, 9(1)(2008), 23-34] introduced a concept of generalized multivalued almost contraction mapping and obtained a fixed point result for this new class of mappings. We extend this notion to multivalued $f-$ almost contraction mappings and prove the existence of coincidence points for such mappings. As a consequence, coincidence point results are obtained for generalized multivalued $f-$ almost nonexpansive mappings which assume closed values only. Related common fixed point theorems are also proved. Our results provide extension as well as substantial generalizations and improvements of several well known results in the existing literature.
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1. Introduction and preliminaries

Let (X,d) be a metric space. We denote by $CB(X)$ and $CL(X)$, the families of all nonempty closed bounded and nonempty closed subsets of X, respectively. For $A,B \in CL(X)$. Set, $E_{A,B} = \{\varepsilon > 0 : A \subseteq N_\varepsilon(B), B \subseteq N_\varepsilon(A)\}$. we define a generalized Hausdorff metric H on $CL(X)$ by

$$H(A,B) = \begin{cases} \inf E_{A,B} & \text{if} \quad E_{A,B} \neq \emptyset \\ \infty & \text{if} \quad E_{A,B} = \emptyset \end{cases}$$

where $N_\varepsilon(A) = \{x \in X : d(x,A) < \varepsilon\}$.
A multivalued mapping $T : X \to CL(X)$ is said to be continuous at point p if $\lim_{n\to\infty} d(x_n,p) = 0$ implies that $\lim_{n\to\infty} H(Tx_n,Tp) = 0$.
Definition 1.1. Let $f : X \longrightarrow X$ and $T : X \longrightarrow CL(X)$. A point x in X is said to be (1) *fixed point of* f if $f(x) = x$; (2) *fixed point of* T if $x \in T(x)$; (3) *coincidence point of a pair* (f,T) if $fx \in Tx$; (4) *common fixed point of a pair* (f,T) if $x = fx \in Tx$.
$F(f), C(f,T)$ and $F(f,T)$ denote set of all fixed points of f, set of all coincidence points of the pair (f,T) and the set of all common fixed points of the pair (f,T), respectively.

In the following definition, M. Berinde and V. Berinde [6] extended the notion of weak contraction from single valued mappings ([4]) to multivalued mappings. For more discussion on single valued weak contraction mappings we refer to [5], [8] and references mentioned therein.
A mapping $T : X \longrightarrow CB(X)$ is said to be multivlaued weak contraction [6] iff there exist two constants $\theta \in (0, 1)$ and $L \geq 0$ such that

$$H(Tx, Ty) \leq \theta d(x, y) + Ld(y, Tx)$$

for every $x, y \in X$.
Recently, Berinde and Păcurar [7] introduced the concept of a generalized multivalued $(\theta, L)-$ strict almost contraction mapping and obtained the following fixed point theorem.
Theorem 1.2. Let (X, d) be a complete metric space, and $T : X \longrightarrow CB(X)$ be a generalized multivalued $(\theta, L)-$ strict almost contraction, that is there exist two constants $\theta \in (0, 1)$ and $L \geq 0$ such that

$$H(Tx, Ty) \leq \theta d(x, y) + L \min\{d(y, Tx), d(x, Ty), d(y, Ty), d(x, Tx)\} \tag{1.1}$$

for every $x, y \in X$. Then $Fix(T) \neq \phi$. Moreover, for any $p \in Fix(T)$, T is continuous at p.
Kamran [13] extended the notion of multivlaued weak contraction mappings for a hybrid pair of $f : X \to X$ and $T : X \to CB(X)$ and obtained the results regarding coincidence points of hybrid pair $\{f, T\}$.
Let f be a self map on X. A mapping $T : X \longrightarrow CL(X)$ is said to be multivalued $f-$ weak contraction ([13]) or multivalued $(f, \theta, L)-$ weak contraction iff there exist two constants $\theta \in (0, 1)$ and $L \geq 0$ such that

$$H(Tx, Ty) \leq \theta d(fx, fy) + Ld(fy, Tx) \tag{1.2}$$

for every $x, y \in X$.
Our following definition extends and generalize the notion of generalized multivalued $(\theta, L)-$ strict almost contraction mapping.
Definition 1.3. Let f be a self map on X. A mapping $T : X \longrightarrow CL(X)$ is said to be generalized multivlaued $(f, \theta, L)-$ almost contraction iff there exist two constants $\theta \in (0, 1)$ and $L \geq 0$ such that

$$H(Tx, Ty) \leq \theta M(x, y) + L \min\{d(fx, Tx), d(fy, Ty), d(fx, Ty), d(fy, Tx)\} \tag{1.3}$$

for all $x, y \in X$, where

$$M(x, y) = \max\left\{d(fx, fy), d(fx, Tx), d(fy, Ty), \frac{d(fx, Ty) + d(fy, Tx)}{2}\right\}.$$

If in above definition, we take $f = I$ (an identity mapping on X), we obtain a generalized version of the concept of generalized multivalued $(\theta, L)-$ strict almost contraction mappings.
Remark 1.4. Suppose that the mapping $T : X \longrightarrow CL(X)$ satisfies

$$H(Tx, Ty) \leq \theta M(x, y) + L \min\{d(x, Tx), d(y, Ty), d(x, Ty), d(y, Tx)\} \tag{1.4}$$

for all $x, y \in X$, where $\theta \in (0,1)$, $L \geq 0$ and

$$M(x,y) = \max\left\{d(x,y), d(x,Tx), d(y,Ty), \frac{d(x,Ty)+d(y,Tx)}{2}\right\}$$

then for any $p \in Fix(T)$, T is continuous at p.
For this let, $\{x_n\}$ be a sequence in X such that $\lim\limits_{n\to\infty} d(x_n,p) = 0$. Replacing x by x_n and y by p in (1.4), we obtain

$$\begin{aligned}
& H(Tp, Tx_n) \\
\leq\ & \theta \max\left\{d(p,x_n), d(x_n,Tx_n), d(p,Tp), \frac{d(x_n,Tp)+d(p,Tx_n)}{2}\right\} \\
& +L\min\{d(p,Tp), d(y_n,Ty_n), d(p,Ty_n), d(y_n,Tp)\} \\
\leq\ & \theta \max\left\{d(p,x_n), d(x_n,Tx_n), \frac{d(x_n,p)+d(p,Tx_n)}{2}\right\} \\
\leq\ & \theta \max\left\{d(p,x_n), (d(x_n,p)+H(Tp,Tx_n)), \frac{d(x_n,p)+H(Tp,Tx_n)}{2}\right\}
\end{aligned}$$

which on taking limit implies that $\lim\limits_{n\to\infty} H(Tx_n, Tp) = 0$.

Definition 1.5. A subset Y of a normed space X is called (5) *$q-$starshaped or starshaped with respect to q* if $\lambda x+(1-\lambda)q \in Y$ for all $x \in Y$ and $\lambda \in [0,1]$; (6) *convex* if $\lambda x + (1-\lambda)y \in Y$ for all $x, y \in Y$ and $\lambda \in [0,1]$.

Definition 1.6. Let f be a self map on a normed space X and $Y \subseteq X$, f is called (7) *affine on* Y if Y is convex and $f(\lambda x+(1-\lambda)y) = \lambda fx+(1-\lambda)fy$ for all $x, y \in Y$ and $\lambda \in [0,1]$; (8) *$q-$affine on* Y if Y is $q-$starshaped and $f(\lambda x+(1-\lambda)q) = \lambda fx+(1-\lambda)q$ for all $x \in Y$ and $\lambda \in [0,1]$.

Definition 1.7. Let $f : Y \longrightarrow Y$ and $T : Y \longrightarrow CL(Y)$. The pair (f,T) is called (9) *commuting* if $Tfx = fTx$ for all $x \in Y$; (10) *weakly compatible* [12] if they commute at their coincidence points, that is, $fTx = Tfx$ whenever $x \in C(f,T)$.

Definition 1.8. Let $f : X \longrightarrow X$, $T : X \longrightarrow CL(X)$ and $Y \subseteq X$. $f - T$ is called (11) *demiclosed at* 0 if whenever a sequence $\{x_n\}$ in Y converges weakly to x_0 in Y and $y_n \in (f-T)x_n$ such that $\{y_n\}$ converges to 0 strongly, then $0 \in (f-T)x_0$. The map f is called (12) *T- weakly commuting* at $x \in X$ if $f^2x \in Tfx$.

If the hybrid pair $\{f,T\}$ is weakly compatible at $x \in C(f,T)$, then f is $T-$weakly commuting at x and hence $f^n(x) \in C(f,T)$, however the converse is not true in general (For detailed discussion on above mentioned notions and their implications, we refer to [3], [9], [11], [12] and references therein).

2. Coincidence and common fixed point results

For practical purposes, a relaxation of boundedness condition is always desired. We obtain common fixed points of generalized multivalued $f-$ nonexpansive mappings which in turn unify and improve comparable coincidence point results for multivalued mappings restricting the range of multivalued mappings to $CL(X)$. We begin with the following result which extends and improves Theorem 4 of [7], Theorem 2.1 of

Al-Thagafi [1], Theorem 2.1 of Al-Thagafi and Shahzad [2], the main result of Jungck [10], and Theorem 2.9 of Kamran [13].

Theorem 2.1. Let X be a metric space, $f : X \longrightarrow X$ and $T : X \longrightarrow CL(X)$ be a generalized multivlaued $(f, \theta, L)-$ almost contraction with $\overline{T(X)} \subseteq f(X)$. If $\overline{T(X)}$ is complete, then $C(f, T) \neq \emptyset$. Moreover, $F(f, T) \neq \emptyset$ if one of the following conditions holds:

(a) For some $x \in C(f, T)$, f is $T-$ weakly commuting at x and $f^2x = fx$.

(b) f and T are weakly compatible on $C(f, T)$, f is continuous, and $\lim\limits_{n\to\infty} f^n x$ exists for some $x \in C(f, T)$.

(c) For some $z \in C(f, T)$, f is continuous at z, and $\lim\limits_{n\to\infty} f^n y = z$ for some $y \in X$.

(d) $f(C(f, T))$ is a singleton subset of $C(f, T)$.

Proof. Let x_0 be in X. Since $Tx_0 \subseteq fX$, pick $x_1 \in X$ such that $fx_1 \in Tx_0$. As $k = \dfrac{1}{\sqrt{\theta}} > 1$, there exists $y_1 \in Tx_1$ such that $d(y_1, fx_1) \leq kH(Tx_1, Tx_0)$. As $T(x_1) \subseteq \overline{T(X)} \subseteq f(X)$, one finds x_2 in X such that $y_1 = fx_2$. Thus $d(fx_2, fx_1) \leq kH(Tx_1, Tx_0)$. Continuing this process, we can construct a sequence $\{x_n\}$ in X such that $fx_{n+1} \in Tx_n \subseteq TX$, for all $n \geq 1$ and $d(fx_{n+1}, fx_n) \leq kH(Tx_n, Tx_{n-1})$. Thus by taking x_n for x and x_{n-1} for y in the inequality (1.3), it follows that

$$\begin{aligned}
& d(fx_{n+1}, fx_n) \\
\leq\ & kH(Tx_n, Tx_{n-1}) \\
\leq\ & \sqrt{\theta}M(x_n, x_{n-1}) + \frac{L}{\sqrt{\theta}} \min\{d(fx_n, Tx_n), d(fx_{n-1}, Tx_{n-1}), \\
& d(fx_n, Tx_{n-1}), d(fx_{n-1}, Tx_n)\}
\end{aligned}$$

where

$$\begin{aligned}
M(x_n, x_{n-1}) &= \max\Big\{d(fx_n, fx_{n-1}), d(fx_n, Tx_n), d(fx_{n-1}, Tx_{n-1}), \\
&\quad \frac{d(fx_n, Tx_{n-1}) + d(fx_{n-1}, Tx_n)}{2}\Big\} \\
&= \max\Big\{d(fx_n, fx_{n-1}), d(fx_n, Tx_n), d(fx_{n-1}, Tx_{n-1}), \\
&\quad \frac{d(fx_{n-1}, Tx_n)}{2}\Big\} \\
&= \max\Big\{d(fx_n, fx_{n-1}), d(fx_n, fx_{n+1}), \frac{d(fx_{n-1}, fx_{n+1})}{2}\Big\} \\
&= \max\{d(fx_n, fx_{n-1}), d(fx_n, fx_{n+1})\}
\end{aligned}$$

which further gives that

$$d(fx_{n+1}, fx_n) \leq \alpha \max\{d(fx_n, fx_{n-1}), d(fx_n, fx_{n+1})\},$$

where $\alpha = \sqrt{\theta} < 1$. Now if for some n,

$$d(fx_n, fx_{n+1}) > d(fx_n, fx_{n-1}),$$

then we have

$$d(fx_{n+1}, fx_n) \leq \alpha d(fx_{n+1}, fx_n)$$

a contradiction. So

$$\begin{aligned} d(fx_n, fx_{n+1}) &\leq \alpha d(fx_{n-1}, fx_n) \\ &\leq \ldots \leq \alpha^n d(fx_0, fx_1). \end{aligned}$$

Now, for any positive integers m and n with $m > n$, we have

$$\begin{aligned} d(fx_m, fx_n) &\leq d(fx_n, fx_{n+1}) + d(fx_{n+1}, fx_{n+2}) + \cdots + d(fx_{m-1}, fx_m) \\ &\leq [\alpha^n + \alpha^{n+1} + \cdots + \alpha^{m-1})]d(fx_0, fx_1) \\ &\leq \frac{\alpha^n}{1-\alpha} d(fx_0, fx_1), \end{aligned}$$

which implies that $\{fx_n\}$ is a Cauchy sequence in $T(X)$. It follows from the completeness of $\overline{T(X)}$ that $fx_n \longrightarrow p \in \overline{T(X)} \subseteq f(X)$. Hence we can find u^* in X such that $fu^* = p$. Now,

$$\begin{aligned} d(p, Tu^*) &\leq d(p, fx_{n+1}) + d(fx_{n+1}, Tu^*) \\ &\leq d(p, fx_{n+1}) + H(Tx_n, Tu^*) \\ &\leq d(p, fx_{n+1}) + \theta \max\Big\{d(fx_n, fu^*), d(fx_n, Tx_n), d(fu^*, Tu^*), \\ &\quad \frac{d(fx_n, Tu^*) + d(fu^*, Tx_n)}{2}\Big\} \\ &\quad + L\min\{d(fx_n, fx_{n+1}), d(fu^*, Tu^*), d(fx_n, Tu^*), d(fu^*, fx_{n+1})\} \\ &= d(p, fx_{n+1}) + \theta \max\Big\{d(fx_n, fu^*), d(fx_n, fx_{n+1}), d(fu^*, Tu^*), \\ &\quad \frac{d(fx_n, Tu^*) + d(fu^*, fx_{n+1})}{2}\Big\} \\ &\quad + L\min\{d(fx_n, fx_{n+1}), d(fu^*, Tu^*), d(fx_n, Tu^*), d(fu^*, fx_{n+1})\} \end{aligned}$$

which on taking limit as $n \to \infty$ gives that

$$\begin{aligned} & d(p, Tu^*) \\ \leq\ & \theta \max\Big\{d(p,p), d(p,p), d(p, Tu^*), \frac{d(p, Tu^*) + d(p,p)}{2}\Big\} \\ & + L\min\{d(p,p), d(p, Tu^*), d(p, Tu^*), d(p,p)\} \end{aligned}$$

which further implies

$$d(p, Tu^*) \leq \theta d(p, Tu^*).$$

Hence $d(p, Tu^*) = 0$ and $fu^* = p \in Tu^*$. Thus $C(f,T) \neq \emptyset$.
(a) Suppose, for $x \in C(f,T)$, f is $T-$ weakly commuting at x; that is, $f^2x \in Tfx$. By the given hypothesis $f^2x = fx$; therefore fx will serve as a common fixed point of f and T.
(b) Suppose that $y = \lim\limits_{n\to\infty} f^n x$ for some $x \in C(f,T)$. Since f is continuous, y is fixed point of f. Also since f and T are weakly compatible on $C(f,T)$, $f^n x \in C(f,T)$ for

all $n \geq 1$, and hence $f^n x \in Tf^{n-1}x$. Consider,

$$\begin{aligned} d(y,Ty) &\leq d(y,f^n x) + d(f^n x,Ty) \\ &\leq d(y,f^n x) + H(Tf^{n-1}x,Ty) \\ &\leq d(y,f^n x) + \theta \max\Big\{ d(f^n x,fy), d(f^n x,Tf^{n-1}x), d(fy,Ty), \\ & \quad \frac{d(f^n x,Ty)+d(fy,Tf^{n-1}x)}{2}\Big\} \\ &+ L\min\{d(f^n x,Tf^{n-1}x), d(fy,Ty), d(f^n x,Ty), d(fy,Tf^{n-1}x)\} \\ &= d(y,f^n x) + \theta \max\Big\{ d(f^n x,y), d(y,Ty), \frac{d(f^n x,Ty)+d(y,f^n x)}{2}\Big\} \\ & \quad L\min\{d(f^n x,Tf^{n-1}x), d(y,Ty), d(f^n x,Ty), d(y,Tf^{n-1}x)\}. \end{aligned}$$

Taking the limit as $n \to \infty$, we obtain $d(y,Ty) \leq \theta d(y,Ty)$ and $d(y,Ty) = 0$ hence $y \in Ty$. Thus $F(f,T) \neq \emptyset$.

(c) Suppose that for some $z \in C(f,T)$, f is continuous at z, and $\lim\limits_{n\to\infty} f^n x = z$ for some $x \in X$. Then $z = fz \in Tz$, and $F(f,T) \neq \emptyset$.

(d) Since $f(C(f,T)) = x$ (say) and $x \in C(f,T)$, this implies that $x = fx \in Tx$. Thus $F(f,T) \neq \emptyset$.

Corollary 2.2. Let X be a metric space, $T : X \longrightarrow CL(X)$ be generalized multivlaued $(\theta, L)-$ almost contraction with $\overline{T(X)} \subseteq X$. Suppose that $\overline{T(X)}$ is complete. Then T has a fixed point.

Corollary 2.2 generalizes the Banach contraction principle, and the Nadler contraction Principle [15]. Theorem 3 of [6] and Theorem 4 of [7] become special cases of our Corollary 2.2.

Let Y be a $q-$starshaped subset of a normed space X, $f : Y \longrightarrow Y$ and $T : Y \longrightarrow CL(Y)$. A pair $\{f,T\}$ satisfies the coincidence point condition on a closed subset A of Y if, whenever $\{x_n\}$ is a sequence in A such that $d(fx_n, Tx_n) \longrightarrow 0$, then $fu \in Tu$ for some $u \in A$. A map T satisfies the fixed point condition on $A \in CL(Y)$ if, whenever $\{x_n\}$ is a sequence in A such that $d(x_n, Tx_n) \longrightarrow 0$, then $u \in Tu$ for some $u \in A$. We also define, $\delta(fy,Tx) = \inf\{d(fy, T_\lambda x) : 0 \leq \lambda \leq 1\}$.

Theorem 2.3. Let Y be a subset of a normed space X, $f : Y \longrightarrow Y$ and $T : Y \longrightarrow CL(Y)$. Suppose that Y is $q-$starshaped, $f(Y) = Y$ [resp. f is $q-$affine on Y], $T(Y)$ is bounded, $\overline{T(Y)}$ is complete, $\overline{T(Y)} \subseteq f(Y)$,the pair $\{f,T\}$ satisfies the coincidence point condition on Y and

$$\begin{aligned} H(Tx,Ty) \leq \max\Big\{ \|fx - fy\|, \delta(fx,Tx), \delta(fy,Ty), \frac{\delta(fx,Ty)+\delta(fy,Tx)}{2}\Big\} \\ + \min\{\delta(fx,Tx), \delta(fy,Ty), \delta(fx,Ty), \delta(fy,Tx)\} \end{aligned} \tag{2.1}$$

for all $x, y \in Y$. Then $C(f,T) \neq \emptyset$. Moreover, $F(f,T) \neq \emptyset$ if one of the conditions $(a)-(d)$ of Theorem (2.1) holds.

Proof. Let $\{\lambda_n\}$ be a sequence in $(0,1)$ such that $\lambda_n \to 1$. For $n \geq 1$, let

$$T_n(x) = T_{\lambda_n}(x) = \lambda_n Tx + (1-\lambda_n)q$$

for all x in Y. As Y is $q-$starshaped, $\overline{T(Y)}$ is complete, $\overline{T(Y)} \subseteq f(Y)$, and $f(Y) = Y$ [resp. f is $q-$affine on Y], we have $\overline{T_n(Y)} \subseteq f(Y)$ and $\overline{T_n(Y)}$ is complete for each $n \geq 1$. Now consider,

$$\begin{aligned}
& H(T_n x, T_n y) \\
= \; & \lambda_n H(Tx, Ty) \\
\leq \; & \lambda_n \max\left\{\|fx - fy\|, \delta(fx, Tx), \delta(fy, Ty), \frac{\delta(fx, Ty) + \delta(fy, Tx)}{2}\right\} \\
& + \lambda_n \min\{\delta(fx, Tx), \delta(fy, Ty), \delta(fx, Ty), \delta(fy, Tx)\} \\
\leq \; & \lambda_n \max\left\{\|fx - fy\|, d(fx, T_n x), d(fy, T_n y), \frac{d(fx, T_n y) + d(fy, T_n x)}{2}\right\} \\
& + \lambda_n \min\{d(fx, T_n x), d(fy, T_n y), d(fx, T_n y), d(fy, T_n x)\}
\end{aligned}$$

for all $x, y \in Y$, which implies that each T_n is a generalized multivalued $(f, \lambda_n, \lambda_n)-$ almost contraction on Y. Hence, from Theorem 2.1 we conclude that $fx_n \in Tx_n = \lambda_n Tx_n + (1 - \lambda_n)q$ for some $x_n \in Y$. As $fx_n = \lambda_n y_n + (1 - \lambda_n)q$ for some $y_n \in Tx_n \subseteq T(Y)$, $T(Y)$ is bounded, $\lambda_n \to 1$ and

$$\|fx_n - y_n\| = (1 - \lambda_n)\,\|q - y_n\| \leq (1 - \lambda_n)(\|q\| + \|y_n\|),$$

so, $\|fx_n - y_n\| \to 0$ and hence $d(fx_n, Tx_n) \leq \|fx_n - y_n\| \to 0$. Since the pair $\{f, T\}$ satisfies the coincidence point condition on Y, there exists a $u \in Y$ such that $fu \in Tu$. Thus $C(f, T) \neq \emptyset$. Using arguments similar to those given in the proof of Theorem 2.1, it can be shown that $F(f, T) \neq \emptyset$ if one of the conditions $(a) - (d)$ of Theorem 2.1 holds.

Clearly an f-nonexpansive multivalued map T satisfies inequality (2.1), so Theorem 2.3 improves and generalizes Corollary 2.5 of Hussain and Jungck [9], Corollaries 3.2, 3.4 of Jungck [11], Theorems 2.2-2.5 of Latif and Tweddle [14], and Theorem 3 due to Rhoades [16].

Corollary 2.4. Let Y be a subset of a normed space X, $f : Y \longrightarrow Y$ and $T : Y \longrightarrow CL(Y)$. Suppose that Y is $q-$starshaped, $f(Y) = Y$ [resp. f is $q-$affine on Y], $\overline{T(Y)}$ is complete, $\overline{T(Y)} \subseteq f(Y)$, and f and T satisfy (2.1) for all $x, y \in Y$. Then $C(f, T) \neq \emptyset$ if one of the following conditions holds.

(e) $T(Y)$ is bounded and $(f - T)(Y)$ is closed.

(f) Y is weakly compact and $f - T$ is demiclosed at 0.

Moreover $F(f, T) \neq \emptyset$ if one of the conditions $(a) - (d)$ of Theorem 2.1 holds.

Proof. (e) As in the proof of Theorem 2.3, $fx_n - y_n \to 0$ as $n \to \infty$ where $y_n \in Tx_n$. As $(f - T)(Y)$ is closed so $0 \in (f - T)(Y)$. Hence the pair $\{f, T\}$ satisfies the coincidence point condition on Y and the result follows from Theorem 2.3.

(f) As in the proof of Theorem 2.3, $fx_n - y_n \to 0$ as $n \to \infty$ where $y_n \in Tx_n$. By the weak compactness of Y, there is a subsequence $\{x_m\}$ of the sequence $\{x_n\}$ such that $\{x_m\}$ converges weakly to $y \in Y$ as $m \to \infty$. Since $f - T$ is demiclosed at 0, we obtain $0 \in (f - T)y$. Hence the pair $\{f, T\}$ satisfies coincidence point condition on Y and the result follows from Theorem 2.3.

Corollary 2.5. Let Y be a subset of a normed space X, and $T : Y \longrightarrow CL(Y)$. Suppose that Y is $q-$starshaped, $T(Y)$ is bounded, $\overline{T(Y)}$ is complete, $\overline{T(Y)} \subseteq Y$, T

satisfies the fixed point condition on Y and

$$\begin{aligned} & H(Tx,Ty) \\ \leq \quad & \max\left\{\|x-y\|, \delta(x,Tx), \delta(y,Ty), \frac{\delta(x,Ty)+\delta(y,Tx)}{2}\right\} \\ & + \min\{\delta(x,Tx), \delta(y,Ty), \delta(x,Ty), \delta(y,Tx)\} \end{aligned}$$

for all $x, y \in Y$. Then T has a fixed point.

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