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CHOICES OF VARIABLE STEPS OF THE CQ ALGORITHM FOR THE SPLIT FEASIBILITY PROBLEM

FENGHUI WANG*, HONG-KUN XU**,[1] AND MENG SU***

*Department of Mathematics, East China University of Science and Technology
Shanghai 200237, China
and
Department of Mathematics, Luoyang Normal University, Luoyang 471022, China
E-mail: wfenghui@gmail.com

**Department of Mathematics, East China University of Science and Technology
Shanghai 200237, China;
Department of Applied Mathematics, National Sun Yat-sen University
Kaohsiung 80424, Taiwan;
and
Department of Mathematics, College of Science, King Saud University
P.O. Box 2455, Riyadh 11451, Saudi Arabia
E-mail: xuhk@math.nsysu.edu.tw

***Penn State University at Erie, The Behrend College
4205 College Drive, Erie, PA 16563-0203, U.S.A.
E-mail: mengsu@psu.edu

Abstract. We consider the CQ algorithm, with choice of steps introduced by Yang (J. Math. Anal. Appl. 302 (2005), 166-179), for solving the split feasibility problem (SFP): find $x \in C$ such that $Ax \in Q$, where C and Q are nonempty closed convex subsets of $\mathbb{R}^n$ and $\mathbb{R}^m$, respectively, and A is an $m \times n$ matrix. We convert the SFP to an equivalent convexly constrained nonlinear system of finding a zero in C of an inverse strongly monotone operator, which enables us to introduce new convergent iterative algorithms. Two restrictive conditions of Yang (i.e., the boundedness of Q and the full column rank of A) are completely removed in our new algorithms.
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[1]Corresponding author.

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