

A FIXED POINT APPROACH TO THE STABILITY OF ADDITIVE FUNCTIONAL INEQUALITIES IN RN-SPACES

CHOONKIL PARK

Department of Mathematics, Hanyang University
Seoul 133-791, Republic of Korea.
E-mail: baak@hanyang.ac.kr

Abstract. Using the fixed point method, we prove the generalized Hyers-Ulam stability of the Cauchy additive functional inequality and of the Cauchy-Jensen additive functional inequality in random normed spaces.

Key Words and Phrases: Additive functional inequality, Fixed point, Generalized Hyers-Ulam stability, Random normed space.

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