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FIXED POINT THEORY FOR CYCLIC BERINDE OPERATORS

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Abstract. Inspired by the considerations in [Kirk, W.A., Srinivasan, P.S., Veeramany, P., *Fixed points for mappings satisfying cyclical contractive conditions*, Fixed Point Theory, **4** (2003), No. 1, 79-89], which were further discussed in [Rus, I.A., *Cyclic representations and fixed points*, Ann. T. Popoviciu Seminar Funct. Eq. Approx. Convexity, **3** (2005), 171-178], we establish the existence and uniqueness of the fixed point for cyclic strict Berinde operators. Following [Rus, I.A., *The theory of a metrical fixed point theorem: theoretical and applicative relevances*, Fixed Point Theory, **9** (2008), No. 2, 541-559], we build a so-called theory of the main result, referring concepts and phenomena like Picard operators, data dependence, limit shadowing, well-posedness of the fixed point problem. A Maia type result for cyclic strict Berinde operators is also given.
Key Words and Phrases: Cyclic almost contraction, cyclic Berinde operator, Picard operator, data dependence, well-posedness of a fixed point problem, limit shadowing.
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1. Introduction

The aim of this paper is to prove a fixed point result for cyclic strict Berinde operators (i.e., cyclic strict almost contractions) and to build a theory of this theorem, by stating and proving several results which refer concepts like good Picard operator, special Picard operator, data dependence, sequences of operators and fixed points, well-posedness of a fixed point problem, limit shadowing property and others. A Maia type result for cyclic strict Berinde operators is also given.

2. Preliminaries

In [12] a class of continuous generalized contractions defined on cyclic structures is studied. The present paper contains a similar approach of another class of operators satisfying a general contraction type condition which does *not* imply the continuity, namely the strict almost contractions.

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The class of almost contractions was introduced in [2] (see also [3], [4], [5], [6], [7], [8], [9], [23]) as follows.

Definition 2.1. *Let (X, d) be a metric space. An operator $f : X \to X$ is called* ***almost contraction*** *if there exist two constants $\delta \in [0, 1)$ and $L \geq 0$ such that:*

$$d(f(x), f(y)) \leq \delta d(x, y) + Ld(y, f(x)), \tag{2.1}$$

for any $x, y \in X$.

In Theorem 1 [2] it is shown that the almost contractions are weakly Picard operators. In the same paper, Theorem 2 adds the following condition on the almost contractions, thus obtaining the uniqueness of the fixed point:

$$d(f(x), f(y)) \leq \delta_u d(x, y) + L_u d(x, f(x)), \tag{2.2}$$

for any $x, y \in X$, where $\delta_u \in [0, 1)$ and $L_u \geq 0$ are constants.

Inspired by this result, in [13] we considered:

Definition 2.2. *Let (X, d) be a metric space. An operator $f : X \to X$ is called* ***strict almost contraction*** *if it satisfies both condition (2.1) and (2.2), with some real constants $\delta \in [0, 1)$, $L \geq 0$ and $\delta_u \in [0, 1)$, $L_u \geq 0$, respectively.*

Terminological remark. Ioan A. Rus suggested that we should call an almost contraction a *Berinde operator* and a strict almost contraction a *strict Berinde operator*. Therefore, from now on we shall follow this suggestion.

The class of strict Berinde operators contains several known classes of contraction type operators, such as Banach, Kannan, Chatterjea, Cirić-Reich-Rus, Zamfirescu and others, see for example [2], [13].

Suggested by the considerations in [11], the following notion was introduced in [17]:

Definition 2.3. *Let X be a nonempty set and $f : X \to X$ an operator. By definition, $X = \bigcup_{i=1}^{m} X_i$ is a* ***cyclic representation*** *of X with respect to f if*

ι) $X_i, i = \overline{1, m}$ are nonempty sets;
$\iota\iota$) $f(X_1) \subset X_2, \ldots, f(X_{m-1}) \subset X_m, f(X_m) \subset X_1$.

Remark 2.1. Under the conditions of Definition 2.3, we consider the Picard iteration associated to f, $\{x_n\}_{n\geq 0}$, defined by

$$x_n = f(x_{n-1}) = f^n(x_0), n \geq 1, \tag{2.3}$$

for some $x_0 \in X$. As $X = \bigcup_{i=1}^{m} X_i$, there is $i_0 \in \{1, \ldots, m\}$ such that $x_0 \in X_{i_0}$. Considering the way $\{x_n\}_{n\geq 0}$ was constructed and in view of $\iota\iota$) in Definition 2.3, it is easy to remark that for each $n \in \mathbb{N}$, there is $i_n \in \{1, \ldots, m\}$ such that

$$x_n \in X_{i_n},\ x_{n+1} \in X_{i_n+1}.$$

This simple remark shall be useful while proving the main result.

In the following we consider $P_{cl}(X)$ the collection of all nonempty closed subsets of a set X. Inspired by the results in [11], [17] and [12], we introduce:

Definition 2.4. *Let (X,d) be a metric space, m a positive integer, $A_1, \ldots, A_m \in P_{cl}(X)$, $Y := \bigcup_{i=1}^{m} A_i$ and $f : Y \to Y$ an operator. If*

ι) $\bigcup_{i=1}^{m} A_i$ *is a cyclic representation of Y w.r.t. f;*

$\iota\iota$) *there exist $\delta \in [0,1)$ and $L \geq 0$ such that*

$$d(f(x), f(y)) \leq \delta d(x,y) + Ld(y, f(x)), \tag{2.4}$$

for any $x \in A_i$, $y \in A_{i+1}$, where $A_{m+1} = A_1$,

then f is a ***cyclic Berinde operator****.*

Having in view Definition 2.2, we can also introduce:

Definition 2.5. *Let (X,d) be a metric space, m a positive integer, $A_1, \ldots, A_m \in P_{cl}(X)$, $Y := \bigcup_{i=1}^{m} A_i$ and $f : Y \to Y$ an operator. If*

ι) $\bigcup_{i=1}^{m} A_i$ *is a cyclic representation of Y w.r.t. f;*

$\iota\iota$) *f is a cyclic Berinde operator with constants $\delta \in [0,1)$ and $L \geq 0$;*

$\iota\iota\iota$) *there exist $\delta_u \in [0,1)$ and $L_u \geq 0$ such that*

$$d(f(x), f(y)) \leq \delta_u d(x,y) + L_u d(x, f(x)), \tag{2.5}$$

for any $x \in A_i$, $y \in A_{i+1}$, where $A_{m+1} = A_1$,

then f is a ***cyclic strict Berinde operator****.*

In order to prove the main result we shall also need the following lemma, proved in [1]:

Lemma 2.1. *Let $\{a_n\}_{n\geq 0}$, $\{b_n\}_{n\geq 0}$ be two sequences of positive real numbers and $q \in (0,1)$ such that:*

ι) $a_{n+1} \leq q a_n + b_n, n \geq 0$;

$\iota\iota$) $b_n \to 0$ *as* $n \to \infty$.

Then $\lim_{n\to\infty} a_n = 0$.

3. The main result

Having in view the definitions above, we state in the following the main result of this paper.

Theorem 3.1. *Let (X,d) be a complete metric space, m a positive integer, $A_1, \ldots, A_m \in P_{cl}(X)$, $Y := \bigcup_{i=1}^{m} A_i$ and $f : Y \to Y$ an operator. Assume that:*

ι) $\bigcup_{i=1}^{m} A_i$ *is a cyclic representation of Y w.r.t. f;*

$\iota\iota$) *f is a cyclic strict Berinde operator with constants $\delta \in [0,1)$, $L \geq 0$ and $\delta_u \in [0,1)$, $L_u \geq 0$, respectively.*

Then:

1) $\bigcap_{i=1}^{m} A_i \neq \emptyset$, *f has a unique fixed point* $x^* \in \bigcap_{i=1}^{m} A_i$ *and the Picard iteration* $\{x_n\}_{n\geq 0}$ *given by (2.3) converges to* x^* *for any starting point* $x_0 \in Y$*;*

2) *the following estimates hold:*

$$d(x_n, x^*) \leq \frac{\delta^n}{1-\delta}\, d(x_0, x_1), n \geq 1; \tag{3.1}$$

$$d(x_n, x^*) \leq \frac{\delta_u}{1-\delta_u}\, d(x_n, x_{n-1}), n \geq 1; \tag{3.2}$$

3) *for any* $x \in Y$*:*

$$d(x, x^*) \leq \frac{1}{1-\delta_u}\, d(x, f(x)). \tag{3.3}$$

Proof. 1) Let $x_0 \in Y = \overset{m}{\underset{i=1}{\cup}} A_i$, so there is some $i_0 \in \{1, \ldots, m\}$ such that $x_0 \in A_{i_0}$. Let $\{x_n\}_{n\geq 0}$ be the Picard iteration of f starting from x_0. For $n \geq 1$ we have that:

$$d(x_n, x_{n+1}) = d(f(x_{n-1}), f(x_n)). \tag{3.4}$$

In view of Remark 2.1, any x_{n-1} and x_n satisfy (2.4), thus (3.4) implies:

$$d(x_n, x_{n+1}) \leq \delta d(x_{n-1}, x_n) + Ld(x_n, f(x_{n-1})),$$

which actually means that

$$d(x_n, x_{n+1}) \leq \delta d(x_{n-1}, x_n), n \geq 1. \tag{3.5}$$

By induction we obtain that

$$d(x_n, x_{n+1}) \leq \delta^n d(x_0, x_1), n \geq 0. \tag{3.6}$$

Thus, using the triangle inequality, for $p \geq 1$ we are led to:

$$d(x_n, x_{n+p}) \leq \delta^n \frac{1-\delta^p}{1-\delta}\, d(x_0, x_1), n \geq 0, \tag{3.7}$$

which, by letting $n \to \infty$, shows that $\{x_n\}_{n\geq 0}$ is a Cauchy sequence in $Y = \overset{m}{\underset{i=1}{\cup}} A_i$. As $A_i \in P_{cl}(X)$, Y is also closed, so the completeness of X implies that of Y. Thus $\{x_n\}_{n\geq 0}$ converges to some $p \in Y$.

On the other hand, due to condition ι), the Picard iteration $\{x_n\}_{n\geq 0}$ has an infinite number of terms in each A_i, $i = \overline{1,m}$. Thus, Y being complete, from each A_i, $i = \overline{1,m}$ we can extract a subsequence which converges to p. As A_i, $i = \overline{1,m}$ are closed sets, it follows that $p \in \overset{m}{\underset{i=1}{\cap}} A_i$.

As $\overset{m}{\underset{i=1}{\cap}} A_i \neq \emptyset$, we may consider the restriction

$$f|_{\overset{m}{\underset{i=1}{\cap}} A_i} : \overset{m}{\underset{i=1}{\cap}} A_i \to \overset{m}{\underset{i=1}{\cap}} A_i,$$

which obviously satisfies the conditions of Theorem 2 in [2], being a strict Berinde operator on the complete subspace $\overset{m}{\underset{i=1}{\cap}} A_i$. Then by Theorem 2 in [2] it has a unique fixed point, say $x^* \in \bigcap_{i=1}^{m} A_i$, that can be obtained by means of the Picard iteration $\{x_n\}_{n\geq 0}$, starting from any $x_0 \in \bigcap_{i=1}^{m} A_i$.

There is still to be proved that $\{x_n\}_{n\geq 0}$ converges to x^* for any starting point in Y. So let $x \in Y$. There exists $j_0 \in \{1, \ldots, m\}$ such that $x \in A_{j_0}$. On the other hand, $x^* \in \bigcap_{i=1}^{m} A_i$, so $x^* \in A_i, i = \overline{1,m}$. Thus by (2.5) we have:

$$d(f(x^*), f(x)) \leq \delta_u d(x, x^*) + L_u d(x^*, f(x^*)) = \delta_u d(x, x^*).$$

By induction we obtain that

$$d(x^*, f^n(x)) \leq \delta_u^n d(x^*, x),$$

so for $x \in Y$ arbitrary we have that

$$f^n(x) \to 0 \text{ as } n \to \infty.$$

Thus the Picard iteration converges to the unique fixed point, starting from any point $x \in Y$.

2) Letting $p \to \infty$ in (3.7) we obtain the a priori estimate (3.1).

We also have, using (2.5), that:

$$\begin{aligned} d(x_n, x^*) &= d(f(x_{n-1}), f(x^*)) \leq \\ &\leq \delta_u d(x^*, x_{n-1}) + L_u d(x^*, f(x^*)) = \delta d(x^*, x_{n-1}) \leq \\ &\leq \delta_u [d(x_{n-1}, x_n) + d(x_n, x^*)], \end{aligned}$$

which leads to the a posteriori estimate (3.2).

3) For any $x \in Y$ we have:

$$d(x, x^*) \leq d(x, f(x)) + d(f(x), f(x^*)) = d(x, f(x)) + d(f(x^*), f(x)).$$

By (2.5), this yields

$$d(x, x^*) \leq d(x, f(x)) + \delta_u d(x, x^*) + L_u d(x^*, f(x^*)) = d(x, f(x)) + \delta_u d(x, x^*),$$

which immediately implies (3.3). □

4. A theory of the main result

In the paper [21] (see also [16], [19], [22]) a model of a so-called *theory of a fixed point theorem* is described. The set of criteria used to analyze the fixed point results refers to concepts like good Picard operator, special Picard operator, data dependence, sequences of operators and fixed points, well-posedness of a fixed point problem, limit shadowing property and others (for details see [14], [15], [21] and the references therein).

Having in view this model, in the following we shall build a theory of Theorem 3.1.

Theorem 4.1. *Let $f : Y \to Y$ be as in Theorem 3.1.*

Then $\sum_{n=0}^{\infty} d(f^n(x), f^{n+1}(x)) < \infty$, for any $x \in Y$, i.e., f is a good Picard operator.

Proof. Let $x_0 \in Y$. By (3.6) in the proof of Theorem 3.1, we know that

$$d(f^n(x_0), f^{n+1}(x_0)) = \delta^n d(x_0, x_1), n \geq 0.$$

Then

$$\sum_{n=0}^{\infty} d(f^n(x_0), f^{n+1}(x_0)) \leq \sum_{n=0}^{\infty} \delta^n d(x_0, x_1) =$$
$$= \lim_{n\to\infty} \sum_{k=0}^{n} \delta^k d(x_0, x_1) = \lim_{n\to\infty} \frac{1-\delta^{n+1}}{1-\delta}\ d(x_0, x_1) =$$
$$= \frac{1}{1-\delta}\ d(x_0, x_1) < \infty,$$

so f is a good Picard operator. □

Theorem 4.2. *Let $f : Y \to Y$ be as in Theorem 3.1.*

Then $\sum\limits_{n=0}^{\infty} d(f^n(x), x^*) < \infty$, for any $x \in Y$, i.e., f is a special Picard operator.

Proof. From the above estimation (3.1) we know that

$$d(x_n, x^*) \leq \frac{\delta^n}{1-\delta}\ d(x_0, x_1), n \geq 0.$$

For $x = x_0 \in X$ we have:

$$\sum_{n\geq 0} d(f^n(x), x^*) = \lim_{n\to\infty} \sum_{k=0}^{n} d(x_k, x^*) =$$
$$= \lim_{n\to\infty} \sum_{k=0}^{n} \frac{\delta^k}{1-\delta} d(x_0, x_1) = \frac{d(x_0, x_1)}{1-\delta} \lim_{n\to\infty} \sum_{k=0}^{n} \delta^k =$$
$$= \frac{1}{(1-\delta)^2}\ d(x_0, x_1) < \infty,$$

so f is a special Picard operator. □

Regarding the data dependence of the fixed point in the case of cyclic strict Berinde operators, we have:

Theorem 4.3. *Let $f : Y \to Y$ be as in the previous Theorem 3.1, with $F_f = \{x_f^*\}$. Let $g : X \to X$ such that*

ι) g has at least a fixed point, say $x_g^* \in F_g$;
$\iota\iota$) there exists $\eta > 0$ such that

$$d(f(x), g(x)) \leq \eta,$$

for any $x \in Y$.

Then

$$d(x_f^*, x_g^*) \leq \frac{\eta}{1-\delta_u}.$$

Proof. By conclusion 4) in Theorem 3.1 we have that:

$$d(x_f^*, x_g^*) \leq \frac{1}{1-\delta_u} d(x_g^*, f(x_g^*)) = \frac{1}{1-\delta_u} d(g(x_g^*), f(x_g^*)),$$

which by $\iota\iota$) yields

$$d(x_f^*, x_g^*) \leq \frac{\eta}{1-\delta_u}.$$

□

A Nadler type result regarding cyclic strict Berinde operators can also be proved:

Theorem 4.4. *Let $f : Y \to Y$ be as in Theorem 3.1 and $f_n : Y \to Y$, $n \in \mathbb{N}$ such that:*

ι) for each $n \in \mathbb{N}$ there exists $x_n^* \in F_{f_n}$;

$\iota\iota$) $f_n \xrightarrow{u} f$ as $n \to \infty$.

Then

$$x_n^* \to x^*, n \to \infty,$$

where $F_f = \{x^*\}$.

Proof. As $\{f_n\}_{n\geq 0}$ converges uniformly to f, there exist $\eta_n \in \mathbb{R}_+$, $n \in \mathbb{N}$ such that $\eta_n \to 0, n \to \infty$ and

$$d(f_n(x), f(x)) \leq \eta_n,$$

for any $x \in Y$. Now applying Theorem 4.3 for each pair f and f_n, $n \in \mathbb{N}$, it follows that

$$d(x_n^*, x^*) \leq \frac{\eta_n}{1-\delta_u}, n \in \mathbb{N}.$$

Since $\eta_n \to 0, n \to \infty$, the conclusion follows immediately. □

Theorem 4.5. *Let $f : Y \to Y$ be as in Theorem 3.1.*

Then the fixed point problem for f is well posed, that is, assuming there exist $z_n \in Y, n \in \mathbb{N}$ such that

$$d(z_n, f(z_n)) \to 0 \text{ as } n \to \infty,$$

this implies that

$$z_n \to x^* \text{ as } n \to \infty,$$

where $F_f = \{x^*\}$.

Proof. By Theorem 3.1, $F_f = \{x^*\}$. Let $z_n \in Y$, $n \in \mathbb{N}$ such that

$$d(z_n, f(z_n)) \to 0, \text{ as } n \to \infty. \tag{4.1}$$

We have that:

$$d(z_n, x^*) \leq d(z_n, f(z_n)) + d(f(z_n), x^*) = d(z_n, f(z_n)) + d(f(x^*), f(z_n)).$$

As $z_n \in Y = \bigcup_{i=1}^{m} A_i$, $n \geq 0$, there is $j_0 \in \{1, \ldots, m\}$ such that $z_n \in A_{j_0}$. We also know that $x^* \in \bigcap_{i=1}^{m} A_i$, therefore we can apply (2.5) in the previous relation, thus obtaining:

$$d(z_n, x^*) \leq d(z_n, f(z_n)) + \delta_u d(x^*, z_n) + L_u d(x^*, f(x^*)).$$

Then

$$d(z_n, x^*) \leq \frac{1}{1-\delta_u} d(z_n, f(z_n)),$$

which by (4.1) obviously implies that

$$d(z_n, x^*) \to 0, \text{ as } n \to \infty.$$

So the fixed point problem for f is well posed. $\square$

Theorem 4.6. *Let $f : Y \to Y$ be as in Theorem 3.1.*

Then f has the limit shadowing property, that is, assuming there exist $z_n \in Y, n \in \mathbb{N}$ such that

$$d(z_{n+1}, f(z_n)) \to 0 \text{ as } n \to \infty,$$

then there exists $x \in Y$ such that

$$d(z_n, f^n(x)) \to 0 \text{ as } n \to \infty.$$

Proof. Let $z_n \in Y$, $n \in \mathbb{N}$ such that

$$d(z_{n+1}, f(z_n)) \to 0, \text{ as } n \to \infty.$$

By Theorem 3.1 we know that $F_f = \{x^*\}$. We shall prove that $\{z_n\}_{n\geq 0}$ converges exactly to x^*. We have that:

$$\begin{aligned} d(z_n, x^*) &\leq d(z_n, f(z_{n-1})) + d(f(z_{n-1}), x^*) \\ &= d(z_n, f(z_{n-1})) + d(f(x^*), f(z_{n-1})). \end{aligned} \tag{4.2}$$

As $z_{n-1} \in Y = \bigcup_{i=1}^{m} A_i$ and $x^* \in \bigcap_{i=1}^{m} A_i$, after a similar reasoning to the one in the proof of Theorem 4.5 we apply (2.5) in the above relation (4.2) and obtain that

$$d(z_n, x^*) \leq d(z_n, f(z_{n-1})) + \delta_u d(x^*, z_{n-1}) + L_u d(x^*, f(x^*)),$$

which implies

$$d(z_n, x^*) \leq d(z_n, f(z_{n-1})) + \delta_u d(z_{n-1}, x^*), n \geq 1. \tag{4.3}$$

As $d(z_n, f(z_{n-1})) \to 0, n \to \infty$ and $\delta_u \in [0, 1)$, by Lemma 2.1 it follows that

$$d(z_n, x^*) \to 0, n \to \infty. \tag{4.4}$$

We also know, from Theorem 3.1, that for any $x_0 \in Y$:

$$f^n(x_0) \to 0, n \to \infty. \tag{4.5}$$

But for any $x_0 \in Y$ we can write:

$$d(z_n, f^n(x_0)) \leq d(z_n, x^*) + d(x^*, f^n(x_0)). \tag{4.6}$$

Now using (4.4) and (4.5) in (4.6) we obtain that, for any $x_0 \in Y$,

$$d(z_n, f^n(x_0)) \to 0, \text{ as } n \to \infty.$$

Thus f has the limit shadowing property. $\square$

A Maia type result for cyclic strict Berinde operators on metric spaces can also be proved:

Theorem 4.7. *Let X be a nonempty set, d and ρ two metrics on X and $f : X \to X$ an operator. We assume that:*

ι) $d(x,y) \le \rho(x,y)$, for any $x, y \in X$;
$\iota\iota$) (X,d) is a complete metric space;
$\iota\iota\iota$) f is continuous with respect to d;
ιv) $f : (X,\rho) \to (X,\rho)$ is a cyclic strict Berinde operator with constants $\delta \in [0,1)$, $L \ge 0$ and $\delta_u \in [0,1)$, $L_u \ge 0$, respectively.

Then:

1) $F_f = \{x^*\}$;
2) the Picard iteration $\{x_n\}_{n\ge 0}$ converges to x^* in (X,d), for any $x_0 \in X$.

Proof. Let $x_0 \in X$.
Using condition ιv) we deduce that $\{f^n(x_0)\}_{n\in\mathbb{N}}$ is a Cauchy sequence in (X,ρ) (see the proof of Theorem 3.1).
By ι) it follows that it is Cauchy in (X,d), as well.
Finally by $\iota\iota$) and $\iota\iota\iota$) it is easy to prove that it actually converges in (X,d) to the unique fixed point of f. □

Remark 4.1. In [13] we have extended the results from [2] regarding strict Berinde operators on metric spaces to a b-metric space setting, also building a theory of the new result.

In the same manner one can extend the main result of this section, namely Theorem 3.1, which is formulated in a metric space setting, to a result for cyclic strict Berinde operators defined on a b-metric space. All the corresponding theory can be build similarly, without major difficulties.

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