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BOUNDARY VALUE PROBLEM FOR FUNCTIONAL DIFFERENTIAL INCLUSIONS ON MANIFOLDS AND FIXED POINTS OF INTEGRAL-TYPE OPERATORS

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Abstract. We investigate the boundary value problem for second order functional differential inclusions of the form $\frac{D}{dt}\dot{m}(t) \in F(t, m_t(\theta), \dot{m}_t(\theta))$ on a complete Riemannian manifold for a C^1-smooth curve $\varphi : [-h, 0] \to M$ as initial value, and a point m_1 that is non-conjugate with $\varphi(0)$ along at least one geodesic of Levi-Civita connection. Here $\frac{D}{dt}$ is the covariant derivative of Levi-Civita connection and $F(t, m(\theta), X(\theta))$ is a set-valued vector field with closed convex values that satisfies upper Caratheodory condition and is given on couples: a continuous curve $m(\theta)$ in M, $\theta \in [-h, 0]$, and a vector field $X(\theta)$ along $m(\theta)$ that is continuous from the left and has limits from the right, under the assumption that F has uniformly quadratic or less than quadratic growth in velocity. Some conditions on certain geometric characteristics and on the distance between $\varphi(0)$ and m_1, under which the problem is solvable, are found. The solution is constructed from a fixed point of an integral-type operator.
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1. Introduction

Let M be a finite-dimensional complete Riemannian manifold and TM be its tangent bundle with natural projection $\pi : TM \to M$. For $I = [-h, 0]$ denote by $D(I, TM)$ the space of couples $(m(\theta), X(\theta))$ where $m(\theta)$ is a continuous curve in M and $X(\theta)$ is a vector field along $m(\theta)$ being continuous from the left and having the limit from the right. Consider a set-valued mapping $F : R \times D(I, TM) \multimap TM$ such that for any $(m(\theta), X(\theta))$ the relation $\pi F(t, m(\theta), X(m(\theta)) = m(0)$ holds. We call such F *a set-valued force field*.

Specify $l > 0$. We investigate the differential inclusion of the form

$$\frac{D}{dt}\dot{m}(t) \in F(t, m_t(\theta), \dot{m}_t(\theta)), \tag{1.1}$$

where as usual for a curve $m(\cdot) : [-h, l] \to M$ and $t \in [0, l]$, we set $m_t(\theta) = m(t + \theta)$ where $\theta \in I$. We suppose that F has either uniformly less than quadratic or quadratic

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growth in velocity on the sets from $D(I, TM)$ (see Definitions 3.1 and 3.2). Also we assume that F satisfies the so called upper Carathéodory condition (see Definition 3.3) and has convex closed values.

The main aim of the paper is to find conditions that guarantee the solvability for some $t_1 \in (0, l)$ of the boundary value problem for (1.1) with right-hand sides as mentioned above, i.e., to find a C^1-curve $m(t)$, $t \in [-h, t_1]$, with absolutely continuous derivative, satisfying (1.1) and such that $m(t) = \varphi(t)$ for $t \in [-h, 0]$ and $m(t_1) = m_1$ where $\varphi(t)$ is a given C^1-curve with $t \in I$ and m_1 is a given point. Note that for such a solution the couple $(m_t(\theta), \dot{m}_t(\theta))$ belongs to $D(I, TM)$ for every $t \in [0, t_1]$.

It should be pointed out that even the two-point boundary value problem for ordinary second order differential equations may not be solvable at all for smooth uniformly bounded single-valued F (see, e.g., [1]) if the boundary points are conjugate along all geodesics of Levi-Civita connection joining them. That is why we suppose that the points $\varphi(0)$ and m_1 are not conjugate along at least one geodesic. We find some conditions on certain geometric characteristics of M, on t_1, and on the distance between $\varphi(0)$ and m_1, under which the problem is solvable. Note that there are examples of second order equations with non-bounded continuous right-hand sides where for a given couple of points the problem is solvable on a sufficiently small time interval but is not solvable on larger intervals. Besides, the problem can be solvable for points rather close to each other and not solvable at all for points with greater distance between them (see examples in [1]).

We construct the solutions of problem under consideration from fixed points of special integral type operators, that act in the space of continuous curves in the tangent space $T_{\varphi(0)}M$.

A boundary value problem similar to that we consider here, was investigated by another method in [2] under assumption that the initial curve $\varphi(t)$ was constant for $t \in [-h, 0]$ (i.e., it was a single point) and the right-hand side satisfied the condition $\|F(t, m(\theta), X(\theta))\| \leq c(1 + \|X(\theta)\|^{\alpha})$ with $\alpha \in [0, 2)$. Existence of solutions was proven for sufficiently small time intervals. Obviously that problem was a particular case of the one considered here.

Note that a single-valued continuous field $\mathfrak{f}$ is a particular case of above-mentioned set-valued fields F. Thus the conditions found here for inclusion (1.1) are also valid for second order functional differential equation $\frac{D}{dt}\dot{m}(t) = \mathfrak{f}(t, m_t, \dot{m}_t)$ with continuous right-hand side. We do not formulate the results for equations separately.

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2. Technical statements

In this section we modify some constructions from [3] for the problem under consideration.

Take $m_0 \in M$, and let $v : [0, 1] \to T_{m_0}M$ be a continuous curve. It is shown in [3] that there exists a unique C^1–curve $m : [0, 1] \to M$ such that $m(0) = m_0$ and the vector $\dot{m}(t)$ is parallel along $m(\cdot)$ to the vector $v(t) \in T_{m_0}M$ at any $t \in [0, 1]$.

Denote the curve $m(t)$ constructed above from the curve $v(t)$, by the symbol $Sv(t)$. Thus we have defined a continuous operator S that sends the Banach space $C^0([0,1], T_{m_0}M)$ of continuous maps (curves) from $[0,1]$ to $T_{m_0}M$ into the Banach manifold $C^1([0,1], M)$ of C^1- maps from $[0,1]$ to M.

Let a point $m_1 \in M$ be non-conjugate to the point $m_0 \in M$ along a geodesic $g(\cdot)$ of the Levi-Civita connection. Everywhere below denote by U_R a ball in $C^0([0,t_1], T_{\varphi(0)}M)$ with center at the origin.

Lemma 2.1. *There exists a ball $U_\varepsilon \subset C^0([0,1], T_{m_0}M)$ of a radius $\varepsilon > 0$ centered at the origin such that for any curve $\hat{u}(t) \in U_\varepsilon \subset C^0([0,1], T_{m_0}M)$ there exists a unique vector $\mathbf{C}_{\hat{u}}$, belonging to a certain bounded neighborhood V of the vector $\dot{g}(0)$ in $T_{m_0}M$, that is continuous in $\hat{u}$ and such that $S(\hat{u} + \mathbf{C}_{\hat{u}})(1) = m_1$*

We introduce the notation $sup_{\mathbf{C}\in V}\|\mathbf{C}\| = C$, where V is from Lemma 2.1.

Remark 2.1. *One can easily show that $\varepsilon < C$. Note that C characterizes the distance between m_0 and m_1 while ε characterizes some properties of the Riemannian geometry on M.*

Lemma 2.2. *Under conditions and notation of Lemma 2.1, let $R > 0$ and $t_1 > 0$ be such that $t_1^{-1}\varepsilon > R$. Then for any curve $u(t) \in U_R \subset C^0([0,t_1], T_{m_0}M)$ there exists a unique vector C_u in a neighborhood $t_1^{-1}V$ of the vector $t_1^{-1}\dot{g}(0)$ in $T_{m_0}M$, continuously depending on u and such that $S(u + C_u)(t_1) = m_1$*

Lemmas 2.1 and 2.2 are modifications of Theorem 3.3 from [3].

For the given curve $\varphi(\cdot)$ we introduce the operator $S_\varphi : C^0([0,t_1], T_{\varphi(0)}M)$ $\to C^0([-h,t_1], M)$, defined as follows: $S_\varphi(v(\cdot))(t) = \varphi(t)$ for $t \in [-h,0]$ and $S_\varphi(v(\cdot))(t) = S(v(\cdot))(t)$ for $t \in [0,t_1]$.

Lemma 2.3. *For specified $t_1 > 0$, $R > 0$ as above and $\varphi(\cdot) \in C^1(I,M)$ all curves $S(v+C_v)_t(\theta)$ with $v(\cdot) \in U_R \subset C^0([0,t_1], T_{\varphi(0)}M)$ take values in a compact set $\Xi \subset M$ that depends on the curve φ, ε and C, introduced above, and does not depend on t_1.*

Proof. Obviously the length of $S_\varphi(v + C_v)(\cdot)$ is a sum of lengths of $\varphi(\cdot)$ and of $S(v + C_v)(\cdot)$. Since the parallel translation preserves the norm of a vector, for any curve $v(\cdot) \in C^0([0,t_1], T_{\varphi(0)}M))$ the length of $S(v + C_v)(\cdot)$ is not greater than $\int_0^{t_1}(R + \|C_v\|)dt \le \int_0^{t_1} t_1^{-1}(\varepsilon + C)dt = \int_0^1(\varepsilon + C)dt = \varepsilon + C$. Denote $N = \sup_{t\in I}\|\dot{\varphi}(t)\|$. It is easy to see that the length of $\varphi(\cdot)$ is not greater than Nh. Hence all curves $\|S_\varphi(v + C_v)_t(\cdot)\|$ lie in a bounded subset Ξ of M. Since M is complete, by Hopf-Rinow theorem any bounded set is compact. □

Lemma 2.4. *Let a real number a satisfy the inequality $0 < a < \frac{\varepsilon}{(\varepsilon+C)^2}$. Then there exists a sufficiently small positive number ϕ such that $(\varepsilon t_1^{-1} - \phi) > 0$ and the inequality $a((\varepsilon t_1^{-1} - \phi) + Ct_1^{-1})^2 < \varepsilon t_1^{-2} - \phi t_1^{-1}$ holds.*

Proof. From the hypothesis of lemma we get $a(\varepsilon t_1^{-1} + Ct_1^{-1})^2 < \varepsilon t_1^{-2}$. From continuity of both sides of this inequality it follows that there exists a sufficiently small number $\phi > 0$ such that $(\varepsilon t_1^{-1} - \phi) > 0$ and the inequality $a((\varepsilon t_1^{-1} - \phi) + Ct_1^{-1})^2 < (\varepsilon t_1^{-1} - \phi)t_1^{-1} = \varepsilon t_1^{-2} - \phi t_1^{-1}$ holds. □

3. Main results

Everywhere below M is a complete Riemannian manifold. Denote $\|X(\cdot)\| = sup_{\theta\in I}\|X(\theta)\|$. Introduce the norm of $F(t,m,X)\in T_mM$ by usual formula:

$$\|F(t,m(\cdot),X(\cdot))\| = sup_{y\in F(t,m(\cdot),X(\cdot))}\|y\|.$$

On $D(I,TM)$ we consider Skorohod's topology (see for example [4], where it is described for the space of functions continuous from the right and having limits from the left, in our case the construction is quite analogous).

Definition 3.1. *We say that F has uniformly less than quadratic growth in velocity if on every set $[0,l]\times\Theta$ with $\Theta\subset D(I,TM)$ such that all curves $\{m(\cdot)\}=\pi\Theta$ belong to a compact set Ω in M, for the couples $(m(\cdot),X(\cdot))$ with every specified $\|X(\cdot)\|$ we have that $\sup_{(t,m(\cdot))\in[0,l]\times\pi\Theta}\ \|F(t,m(\cdot),X(\cdot))\|$ is finite and the relation*

$$\lim_{\|X(\cdot)\|\to\infty}\frac{\sup_{(t,m(\cdot))\in[0,l]\times\pi\Theta}\ \|F(t,m(\cdot),X(\cdot))\|}{\|X(\cdot)\|^2}=0 \tag{3.1}$$

takes place.

Obviously the uniformly bounded force field is a particular case of that with uniformly less than quadratic growth.

Definition 3.2. *We say that F has uniformly quadratic growth in velocity, if on every set $[0,l]\times\Theta$ with $\Theta\subset D(I,TM)$ such that all curves $\{m(\cdot)\}=\pi\Theta$ belong to a compact set Ω in M, for the couples $(m(\cdot),X(\cdot))$ with every specified $\|X(\cdot)\|$ we have that $\sup_{(t,m(\cdot))\in[0,l]\times\pi\Theta}\ \|F(t,m(\cdot),X(\cdot))\|$ is finite and there exists a positive number $\delta=\delta(\Omega)$ such that*

$$\lim_{\|X(\cdot)\|\to\infty}\frac{\sup_{(t,m(\cdot))\in[0,l]\times\pi\Theta}\ \|F(t,m(\cdot),X(\cdot))\|}{\|X(\cdot)\|^2}=\delta. \tag{3.2}$$

Definition 3.3. *We say that $F(t,m(\theta),X(\theta)))$ satisfies upper Carathéodory conditions if:*

(1) for every couple $(m(\cdot),X(\cdot)))\in D(I,TM)$ the map $F(\cdot,m(\cdot),X(\cdot))):[0,l]\multimap T_mM$ is measurable,

(2) for almost all $t\in I$ the map $F(t,\cdot,\cdot):D(I,TM)\multimap TM$ is upper semicontinuous.

Consider a curve $\varphi(\theta)\in C^1(I,M)$ and a point $m_1\in M$.

Theorem 3.4. *Let $\varphi(0)$ and m_1 be not conjugate along at least one geodesic of Levi-Civita connection joining them and let $F(t,m(\cdot),X(\cdot))$ satisfy the upper Caratheodory condition, have convex closed values and have uniformly less than quadratic growth in velocity. Then for a sufficiently small $t_1>0$ there exists a solution $m(t)$ of (1.1), for which $m(t)=\varphi(t)$ for $t\in I$, and $m(t_1)=m_1$.*

Proof. Since $\varphi(0)$ and m_1 are not conjugate along a geodesic of Levi-Civita connection, the numbers ε and C from Lemma 2.1 are well-posed. Denote by Θ the subset in $D(I,TM)$ such that all curves from $\pi\Theta$ belong to the compact Ξ from Lemma 2.3.

Consider a continuous curve $v : [0,t_1] \to T_{\varphi(0)}M$. Construct the C^1-curve $\gamma(t) = S_\varphi v(t)$ for $t \in [0,t_1]$.

Note that the vector filed $\dot\gamma(t)$ along $\gamma(t)$ is discontinuous at $t = 0$ but the couple $(\gamma_t(\cdot), \dot\gamma_t(\cdot))$ belongs to $D(I,TM)$ for all $t \in [0,l]$. Hence the set-valued vector field $F(t, \gamma_t(\cdot), \dot\gamma_t(\cdot))$ is well-posed for all $t \in [0,t_1]$.

Denote by Γ the operator of parallel translation of vectors along $\gamma(\cdot)$ at the point $\gamma(0) = \varphi(0)$. Apply operator Γ to all sets $F(t, \gamma_t(\theta), \dot\gamma_t(\theta))$ along $\gamma(\cdot)$. As a result for any $v(\cdot) \in C^0(I, T_{m_0}M)$ we obtain a set-valued map $\Gamma F S_\varphi v : [0,t_1] \to T_{m_0}M$ that has convex values. It follows from the results of [5] that this map satisfies upper Carathéodory conditions. Denote by $\mathcal{P}\Gamma F S_\varphi v$ the set of all measurable selections of $\Gamma F S_\varphi v$ (such selections do exist, see e.g., [6]). Define the set-valued operator $\int \mathcal{P}\Gamma F S_\varphi : C^0([0,t_1], T_{m_0}M) \multimap C^0([0,t_1], T_{m_0}M)$ by the formula

$$\int \mathcal{P}\Gamma F S_\varphi = \{\int_0^t f(\tau)d\tau | f(\cdot) \in \mathcal{P}\Gamma F S_\varphi\}.$$

In complete analogy with [5], it can be shown that $\int \mathcal{P}\Gamma F S_\varphi$ is upper semicontinuous, has convex values and sends bounded sets from $C^0([0,t_1], T_{\varphi(0)}M)$ into compact ones.

Introduce $N = \sup_{t\in I} \|\dot\varphi(t)\|$. Consider $a < \frac{\varepsilon}{(\varepsilon+C)^2}$. From (3.1) it is easy to see that there exists a positive number Q such that the following inequalities hold: $N < Q$ and for C^1-curves $m(\cdot)$ and $n(\cdot)$ from $\pi\Theta$ such that $\|\dot m(\cdot)\| > \|\dot n(\cdot)\|$ and $\|\dot m(\cdot)\| > Q$, we have $\|F(t, n(\cdot), \dot n(\cdot))\| < a\|\dot m(\cdot)\|^2$. Take sufficiently small positive number t_1 such that the following conditions are satisfied: $t_1 \in [0,l]$ and $t_1^{-1}\varepsilon - \phi > Q$, where ϕ is a number from Lemma 2.4. Consider a ball $U_R \subset C^0([0,t_1], T_{m_0}M)$, where $R = t_1^{-1}\varepsilon - \phi$. Since $\varepsilon t_1^{-1} > R$, by Lemma 2.2 for any $v(\cdot) \in U_R$ the vector C_v is well-posed. Thus we can introduce the operator $Z : U_R \multimap C^0([0,t_1], T_{m_0}M)$ by formula:

$$Z(v) = \int \mathcal{P}\Gamma F S_\varphi (v + C_v).$$

As well as $\int \mathcal{P}\Gamma F S_\varphi v$, this operator is upper semi-continuous, convex-valued and sends bounded sets from $C^0([0,t_1], T_{m_0}M)$ into compact ones (see [1]). Since $t_1^{-1}\varepsilon - \phi > Q$ and parallel translation preserves the norms of vectors, from the construction of S_φ and from Lemma 2.4 we derive that for any $v(\cdot) \in U_R$ and $t \in [0,t_1]$ the estimate

$$\|F(t, S_\varphi(v + C_v)_t(\theta), \frac{d}{d\theta} S_\varphi(v + C_v)_t(\theta))\| < a((\varepsilon t_1^{-1} - \phi) + C t_1^{-1})^2 < (\varepsilon t_1^{-2} - \phi t_1^{-1})$$

holds. Since parallel translation preserves the norms of vectors, from the last inequality it follows that

$$\|Z(v + C_v)\| = \| \int \mathcal{P}\Gamma F S_\varphi (v + C_v)\|_{C^0([0,t_1], T_{m_0}M)} \le (t_1^{-1}\varepsilon - \phi) = R.$$

Thus Z sends the ball U_R into itself and from the Bohnenblust-Karlin fixed point theorem (see, e.g., [6, 7]) it follows that it has a fixed point $u(\cdot) \in U_R$, i.e. $u(\cdot) \in Zu(\cdot)$. Let us show that $m(t) = S_\varphi(u(t) + C_u)$ is the desired solution. By construction we have $m(\cdot) = \varphi(\cdot)$ for $t \in [-h, 0]$ and $m(t_1) = m_1$.

Note that $\dot u(\cdot)$ is a selection of $\Gamma F(t, S_\varphi(u + C_u)_t(\theta), \frac{d}{d\theta} S_\varphi(u + C_u)_t(\theta))$ since u is a fixed point of Z. In other words, the inclusion $\dot u(t) \in \Gamma F(t, S_\varphi(u + C_u)_t(\theta), \frac{d}{d\theta} S_\varphi(u +$

$C_u)_t(\theta))$ holds for all points t at which the derivative exists. Using the properties of the covariant derivative and the definition of u, one can show that $\dot{u}(t)$ is parallel to $\frac{D}{dt}\dot{m}(t)$ along $m(\cdot)$ and $\Gamma F(t, S_\varphi(u + C_u)_t(\theta), \frac{d}{d\theta} S_\varphi(u + C_u)_t(\theta))$ is parallel to $F(t, m_t(\theta), \dot{m}_t(\theta))$. Hence, $\frac{D}{dt}\dot{m}(t) \in F(t, m(t), \dot{m}(t))$. □

Corollary 3.5. *The assertion of Theorem 3.4 remains true if F does not have uniformly less than quadratic growth in velocity but (3.1) is fulfilled on Θ such that all curves from $\pi\Theta$ belong to the compact Ξ from Lemma 2.3 constructed for φ and m_1 considered in Theorem 3.4.*

Indeed, in the proof of Theorem 3.4 we dealt with such Θ only.

Theorem 3.6. *Let $F(t, m(\cdot), X(\cdot))$ satisfy the upper Carathéodory condition, have convex closed values and have uniformly quadratic growth in velocity. Let the estimate $\delta < \frac{\varepsilon}{(\varepsilon+C)^2}$ hold. Then for a sufficiently small $t_1 > 0$ there exists a solution $m(t)$ of (1.1) such that $m(\cdot) = \varphi(\cdot)$ for $t \in I$ and $m(t_1) = m_1$.*

Proof. Consider numbers ε, C, N and set Θ defined as in the proof of Theorem 3.4. Consider the positive number a such that $\delta < a < \frac{\varepsilon}{(\varepsilon+C)^2}$. From condition (3.2) it follows that there exists a number $Q > 0$, such that the following conditions holds: $Q > N$ and for every $m(\cdot)$ and $n(\cdot)$ from Θ such that $\|\dot{m}(t)\| > \|\dot{n}(t)\|$ and $\|\dot{m}(t)\| > Q$, we have $\|F(t, n(t), \dot{n}(t))\| < a\|\dot{m}(t)\|^2$. For a sufficiently small positive t_1 the following conditions are satisfied: $t_1 \in [0, l]$ and $t_1^{-1}\varepsilon - \phi > Q$, where ϕ is a number from Lemma 2.4. Consider a ball $U_R \subset C^0([0, t_1], T_{m_0}M)$, where $R = t_1^{-1}\varepsilon - \phi$. As in the proof of Theorem 3.4 we can use the operator $Z : U_R \multimap C^0([0, t_1], T_{\varphi(0)}M)$ and one can easily show that this operator is upper semi-continuous, convex-valued and sends bounded sets from $C^0([0, t_1], T_{\varphi(0)}M)$ into compact ones (see [1]).

Since parallel translation preserves the norms of vectors, from the construction of S and from the hypothesis we derive that for any $v(\cdot) \in U_R$ and $t \in [0, t_1]$ the estimate

$$\|F(t, S_\varphi(v + C_v)_t(\theta), \frac{d}{d\theta} S_\varphi(v + C_v)_t(\theta))\| < a((\varepsilon t_1^{-1} - \phi) + C t_1^{-1})^2 < (\varepsilon t_1^{-2} - \phi t_1^{-1})$$

holds. Since parallel translation preserves the norms of vectors, from the last inequality it follows that

$$\|Z(v + C_v)\| = \| \int \mathcal{P}\Gamma F S_\varphi(v + C_v)\|_{C^0([0,t_1], T_{\varphi(0)}M)} \leq (t_1^{-1}\varepsilon - \phi) = R.$$

Thus Z sends the ball U_R into itself and from the Bohnenblust-Karlin fixed point theorem (see, e.g., [6, 7]) it follows that it has a fixed point $u(\cdot) \in U_R$, i.e., $u(\cdot) \in Zu(\cdot)$. It is obvious that, as in the proof of Theorem 3.4, $m(t) = S_\varphi(u(t) + C_u)$ is the desired solution. □

In analogy to Theorem 3.4, the following corollary to Theorem 3.6 takes place:

Corollary 3.7. *The assertion of Theorem 3.6 remains true if F does not have uniformly quadratic growth in velocity but (3.2) is fulfilled on Θ such that all curves from $\pi\Theta$ belong to the compact Ξ from Lemma 2.3 constructed for φ and m_1 considered in Theorem 3.6.*

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