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BOUNDARY VALUE PROBLEM FOR FUNCTIONAL DIFFERENTIAL INCLUSIONS ON MANIFOLDS AND FIXED POINTS OF INTEGRAL-TYPE OPERATORS

P.E. KISELYOVA* AND P.S. ZYKOV**,[1]

* Physics and Mathematics Faculty, Kursk State University
ul. Radishcheva, 33, 305416 Kursk, Russia.
E-mails: poliny@kursknet.ru and petya39b@gmail.com

Abstract. We investigate the boundary value problem for second order functional differential inclusions of the form $\frac{D}{dt}\dot{m}(t) \in F(t, m_t(\theta), \dot{m}_t(\theta))$ on a complete Riemannian manifold for a C^1-smooth curve $\varphi : [-h, 0] \to M$ as initial value, and a point m_1 that is non-conjugate with $\varphi(0)$ along at least one geodesic of Levi-Civita connection. Here $\frac{D}{dt}$ is the covariant derivative of Levi-Civita connection and $F(t, m(\theta), X(\theta))$ is a set-valued vector field with closed convex values that satisfies upper Caratheodory condition and is given on couples: a continuous curve $m(\theta)$ in M, $\theta \in [-h, 0]$, and a vector field $X(\theta)$ along $m(\theta)$ that is continuous from the left and has limits from the right, under the assumption that F has uniformly quadratic or less than quadratic growth in velocity. Some conditions on certain geometric characteristics and on the distance between $\varphi(0)$ and m_1, under which the problem is solvable, are found. The solution is constructed from a fixed point of an integral-type operator.
Key Words and Phrases: Fixed points, integral operators, Riemannian manifolds, boundary value problem, second order functional differential inclusions, non-conjugate points.
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[1]Corresponding author.

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