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JORDAN $*-$HOMOMORPHISMS BETWEEN UNITAL C^*-ALGEBRAS: A FIXED POINT APPROACH

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Abstract. Let A, B be two unital C^*-algebras. By using fixed pint methods, we prove that:

a) Every almost unital almost linear mapping $h : A \longrightarrow B$ which satisfies $h(2^n uy + 2^n yu) = h(2^n u)h(y) + h(y)h(2^n u)$ for all $u \in U(A)$, all $y \in A$, and all $n = 0, 1, 2, ...$, is a Jordan homomorphism.

b) For a unital C^*-algebra A of real rank zero, every almost unital almost linear continuous mapping $h : A \longrightarrow B$ is a Jordan homomorphism when $h(2^n uy + 2^n yu) = h(2^n u)h(y) + h(y)h(2^n u)$ holds for all $u \in I_1(A_{sa})$, all $y \in A$, and all $n = 0, 1, 2, ...$.

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1. Introduction

B.E. Johnson (Theorem 7.2 of [3]) investigated almost algebra $*-$homomorphisms between Banach $*-$algebras: Suppose that U and B are Banach $*-$algebras which satisfy the conditions of (Theorem 3.1 of [3]). Then for each positive ϵ and K there is a positive δ such that if $T \in L(U, B)$ with $\|T\| < K, \|T^\vee\| < \delta$ and $\|T(x^*)^* - T(x)\| < \delta\|x\|(x \in U)$, then there is a $*-$homomorphism $T' : U \longrightarrow B$ with $\|T - T'\| < \epsilon$. Here $L(U, B)$ is the space of bounded linear maps from U into B, and $T^\vee(x, y) = T(xy) - T(x)T(y)(x, y \in U)$. See [3] for details.

We recall a fundamental result in fixed point theory (see [6, 7, 8] for more details).

Theorem 1.1. *(The alternative of fixed point* [2]*). Suppose that we are given a complete generalized metric space (Ω, d) and a strictly contractive mapping $T : \Omega \to \Omega$ with Lipschitz constant L. Then for each given $x \in \Omega$, either*

$d(T^m x, T^{m+1}x) = \infty$ *for all* $m \geq 0$,

or other exists a natural number m_0 such that

$d(T^m x, T^{m+1}x) < \infty$ *for all* $m \geq m_0$;

the sequence $\{T^m x\}$ is convergent to a fixed point y^* of T;

y^* is the unique fixed point of T in the set $\Lambda = \{y \in \Omega : d(T^{m_0}x, y) < \infty\}$;

$d(y, y^*) \leq \frac{1}{1-L} d(y, Ty)$ *for all* $y \in \Lambda$.

Throughout this paper, let A be a unital C^*-algebra with unit e, and B a unital C^*-algebra. Note that a linear mapping $h : A \to B$ is a Jordan $*-$homomorphism if h is $*-$preserving and

$$h(ab + ba) = h(a)h(b) + h(b)h(a)$$

for all $a, b \in A$. Let $U(A)$ be the set of unitary elements in A, $A_{sa} := \{x \in A | x = x^*\}$, and $I_1(A_{sa}) = \{v \in A_{sa} | \|v\| = 1, v \in Inv(A)\}$. Recently, C. Park, D.-H. Boo and J.-S. An [5] investigated almost homomorphisms between unital C^*-algebras. In this paper, by using the fixed point methods, we prove that every almost unital almost linear mapping $h : A \longrightarrow B$ is a Jordan homomorphism when $h(2^n uy + 2^n yu) = h(2^n u)h(y) + h(y)h(2^n u)$ holds for all $u \in U(A)$, all $y \in A$, and all $n = 0, 1, 2, ...$, and that for a unital C^*-algebra A of real rank zero (see [1]), every almost unital almost linear continuous mapping $h : A \longrightarrow B$ is a Jordan homomorphism when $h(2^n uy + 2^n yu) = h(2^n u)h(y) + h(y)h(2^n u)$ holds for all $u \in I_1(A_{sa})$, all $y \in A$, and all $n = 0, 1, 2, ...$.

Note that a unital C^*-algebra is of real rank zero, if the set of invertible self–adjoint elements is dense in the set of self–adjoint elements (see [1]). We denote the algebric center of algebra A by $Z(A)$.

2. Main results

We start our work with the following theorem which investigate almost Jordan $*-$homomorphisms between unital C^*-algebras.

Theorem 2.1. *Let $f : A \to B$ be a mapping such that $f(0) = 0$ and that*

$$f(2^n uy + 2^n yu) = f(2^n u)f(y) + f(y)f(2^n u) \tag{2.1}$$

for all $u \in U(A), y \in A, n \in \mathbb{Z}_+$. There exists a function $\phi : A^3 \to [0, \infty)$ such that:

(i)

$$\|\mu f(\frac{x+y}{2}) + \mu f(\frac{x-y}{2}) - f(\mu x) + f(u^*) - f(u)^*\| \le \phi(x, y, u) \tag{2.2}$$

for all $\mu \in \mathbb{T}$ and all $x, y \in A$, $u \in (U(A) \cup \{0\})$.

(ii) There exists an $L < 1$ such that

$$\phi(x, y, u) \le 2L\phi(\frac{x}{2}, \frac{y}{2}, \frac{u}{2})$$

for all $x, y, u \in A$.

If $\lim_n \frac{f(2^n e)}{2^n} \in U(B) \cap Z(B)$ then the mapping $f : A \to B$ is a Jordan $*-$homomorphism.

Proof. It follows from $\phi(x, y, a) \le 2L\phi(\frac{x}{2}, \frac{y}{2}, \frac{a}{2})$ that

$$\lim_j 2^{-j}\phi(2^j x, 2^j y, 2^j a) = 0 \tag{2.3}$$

for all $x, y, a \in A$.

Put $\mu = 1, y = u = 0$ in (2.2) to obtain

$$\|2f(\frac{x}{2}) - f(x)\| \le \phi(x, 0, 0) \tag{2.4}$$

for all $x \in A$. Hence,

$$\|\frac{1}{2}f(2x) - f(x)\| \leq \frac{1}{2}\phi(2x, 0, 0) \leq L\phi(x, 0, 0) \tag{2.5}$$

for all $x \in A$.
Consider the set $X := \{g \mid g : A \to B\}$ and introduce the generalized metric on X:

$$d(h, g) := inf\{C \in \mathbb{R}^+ : \|g(x) - h(x)\| \leq C\phi(x, 0, 0)\forall x \in A\}.$$

It is easy to show that (X, d) is complete. Now we define the linear mapping $J : X \to X$ by

$$J(h)(x) = \frac{1}{2}h(2x)$$

for all $x \in A$. By Theorem 3.1 of [2],

$$d(J(g), J(h)) \leq Ld(g, h)$$

for all $g, h \in X$.
It follows from (2.5) that

$$d(f, J(f)) \leq L.$$

By Theorem 1.1, J has a unique fixed point in the set $X_1 := \{h \in X : d(f, h) < \infty\}$. Let H be the fixed point of J. H is the unique mapping with

$$H(2x) = 2H(x)$$

for all $x \in A$ satisfying there exists $C \in (0, \infty)$ such that

$$\|H(x) - f(x)\| \leq C\phi(x, 0, 0)$$

for all $x \in A$. On the other hand we have $\lim_n d(J^n(f), D) = 0$. It follows that

$$\lim_n \frac{1}{2^n}f(2^n x) = H(x) \tag{2.6}$$

for all $x \in A$. It follows from (2.2), (2.3) and (2.6) that

$$\begin{aligned}
&\|H(\frac{x+y}{2}) + H(\frac{x-y}{2}) - H(x)\| \\
&= \lim_n \frac{1}{2^n}\|f(2^{n-1}(x+y)) + f(2^{n-1}(x-y)) - f(2^n x)\| \\
&\leq \lim_n \frac{1}{2^n}\phi(2^n x, 2^n y, 0) \\
&= 0
\end{aligned}$$

for all $x, y \in A$. So

$$H(\frac{x+y}{2}) + H(\frac{x-y}{2}) = H(x)$$

for all $x, y \in A$. Put $z = \frac{x+y}{2}, t = \frac{x-y}{2}$ in above equation, we get

$$H(z) + H(t) = H(z + t) \tag{2.7}$$

for all $z, t \in A$. Hence, H is Cauchy additive. By putting $y = x, u = 0$ in (2.2), we have

$$\|\mu f(\frac{2x}{2}) - f(\mu x)\| \leq \phi(x, x, 0)$$

for all $x \in A$. It follows that

$$\|H(2\mu x) - 2\mu H(x)\| = \lim_m \frac{1}{2^m}\|f(2\mu 2^m x) - 2\mu f(2^m x)\| \leq \lim_m \frac{1}{2^m}\phi(2^m x, 2^m x, 0) = 0$$

for all $\mu \in \mathbb{T}$, and all $x \in A$. One can show that the mapping $H : A \to B$ is $\mathbb{C}-$linear. On the other hand by using (2.2), we have

$$\begin{aligned}
&\|H(u^*) - (H(u))^*\| = \lim_n \|\frac{1}{2^n}f(2^n u^*) - \frac{1}{2^n}(f(2^n u))^*\| \\
&\quad \leq \lim_n \frac{1}{2^n}\phi(0,0,2^n u) \leq \lim_n \frac{1}{2^n}\phi(0,0,2^n u) \\
&\quad = 0 \qquad (2.8)
\end{aligned}$$

for all $u \in U(A)$. Now, let $x \in A$. By Theorem 4.1.7 of [4], x is a finite linear combination of unitary elements, i.e., $x = \sum_{j=1}^n c_j u_j$ $(c_j \in \mathbb{C}, u_j \in U(A))$. Since H is $C-$linear, it follows from (2.8) that

$$H(x^*) - H(x)^* = H(\sum_{j=1}^n c_j u_j^*) - H(\sum_{j=1}^n c_j u_j)^* = 0.$$

Hence, H is $*-$preserving. Now, we show that H is a Jordan homomorphism. To this end, let $u \in U(A), y \in A$. Then by linearity of H and (2.1), we have

$$\begin{aligned}
H(uy + yu) &= \lim_n \frac{f(2^n uy + 2^n yu)}{2^n} = \lim_n [\frac{f(2^n u)}{2^n} f(y) + f(y)\frac{f(2^n u)}{2^n}] \\
&= H(u)f(y) + f(y)H(u) \qquad (2.9)
\end{aligned}$$

for all $u \in U(A)$, all $y \in A$. Since H is additive, then by (2.9), we have

$$2^n H(uy + yu) = H(u(2^n y) + (2^n y)u) = H(u)f(2^n y) + f(2^n y)H(u)$$

for all $u \in U(A)$ and all $y \in A$. Hence,

$$H(uy + yu) = \lim_n [H(u)\frac{f(2^n y)}{2^n} + \frac{f(2^n y)}{2^n}H(u)] = H(u)H(y) + H(y)H(u) \qquad (2.10)$$

for all $u \in U(A)$ and all $y \in A$.

Now, let $x \in A$. Then there are $n \in \mathbb{N}, c_j \in \mathbb{C}, u_j \in U(A), 1 \leq j \leq n$, such that $x = \sum_{j=1}^n c_j u_j$, it follows from (2.10) that

$$\begin{aligned}
H(xy + yx) &= H(\sum_{j=1}^n c_j u_j y + \sum_{j=1}^n c_j y u_j) = \sum_{j=1}^n c_j H(u_j y + y u_j) \\
&= \sum_{j=1}^n c_j (H(u_j y) + H(y u_j)) = \sum_{j=1}^n c_j (H(u_j)H(y) + H(y)H(u_j)) \\
&= H(\sum_{j=1}^n c_j u_j)H(y) + H(y)H(\sum_{j=1}^n c_j u_j) \\
&= H(x)H(y) + H(y)H(x)
\end{aligned}$$

for all $y \in A$. This means that H is a Jordan homomorphism.

On the other hand, we have

$$H(e)=\lim_n \frac{f(2^n e)}{2^n}\in U(B)\cap Z(B).$$

Hence, it follows from (2.9) and (2.10) that

$$\begin{aligned}2H(e)H(y) &= H(e)H(y)+H(y)H(e)=H(ye+ey)\\ &= H(e)f(y)+f(y)H(e)=2H(e)f(y)\end{aligned}$$

for all $y\in A$. Since $H(e)$ is invertible, then $H(y)=f(y)$ for all $y\in A$. This completes the proof of theorem. □

Corollary 2.2. *Let $p\in(0,1),\theta\in[1,\infty)$ be real numbers. Let $f:A\to B$ be a mapping such that $f(0)=0$. Let*

$$f(2^n uy+2^n yu)=f(2^n u)f(y)+f(y)f(2^n u)$$

for all $u\in U(A), y\in A, n\in\mathbb{Z}_+$.

(i)

$$\|\mu f(\frac{x+y}{2})+\mu f(\frac{x-y}{2})-f(\mu x)+f(u^*)-f(u)^*\|\le\theta(\|x\|^p+\|y\|^p+\|u\|^p)$$

for all $\mu\in\mathbb{T}$ and all $x,y\in A,\ u\in(U(A)\cup\{0\})$.

If $\lim_n\frac{f(2^n e)}{2^n}\in U(B)\cap Z(B)$ then the mapping $f:A\to B$ is a Jordan $*-$homomorphism.

Proof. The conclusion follows from Theorem 2.1, by putting $\phi(x,y,u):=\theta(\|x\|^p+\|y\|^p+\|u\|^p)$ all $x,y,u\in A$ and $L=2^{p-1}$. □

Theorem 2.3. *Let A be a C^*-algebra of real rank zero. Let $f:A\to B$ be a mapping such that $f(0)=0$ and*

$$f(2^n uy+2^n yu)=f(2^n u)f(y)+f(y)f(2^n u) \tag{2.11}$$

for all $u\in I_1(A_{sa}), y\in A, n\in\mathbb{Z}_+$. There exists a function $\phi:A^3\to[0,\infty)$ such that

(i)

$$\|\mu f(\frac{x+y}{2})+\mu f(\frac{x-y}{2})-f(\mu x)+f(u^*)-f(u)^*\|\le\phi(x,y,u) \tag{2.12}$$

for all $\mu\in\mathbb{T}$ and all $x,y\in A,\ u\in(I_1(A_{sa})\cup\{0\})$.

(ii) There exists an $L<1$ such that $\phi(x,y,u)\le 2L\phi(\frac{x}{2},\frac{y}{2},\frac{u}{2})$ for all $x,y,u\in A$.

If $\lim_n\frac{f(2^n e)}{2^n}\in U(B)\cap Z(B)$ then the mapping $f:A\to B$ is a Jordan $*-$homomorphism.

Proof. By the same reasoning as the proof of Theorem 2.1, there exists a unique $\mathbb{C}-$linear mapping $H:A\to B$ satisfying (2.6). It follows from (2.11) that

$$\begin{aligned}H(uy+yu)=\lim_n\frac{f(2^n uy+2^n yu)}{2^n}&=\lim_n[\frac{f(2^n u)}{2^n}f(y)+f(y)\frac{f(2^n u)}{2^n}]\\ &=H(u)f(y)+f(y)H(u)\end{aligned} \tag{2.13}$$

for all $u\in I_1(A_{sa})$, and all $y\in A$. By additivity of H and (2.13), we obtain that

$$2^n H(uy+yu)=H(u(2^n y)+(2^n y)u)=H(u)f(2^n y)+f(2^n y)H(u)$$

for all $u \in I_1(A_{sa})$ and all $y \in A$. Hence,

$$H(uy + yu) = \lim_n [H(u)\frac{f(2^n y)}{2^n} + \frac{f(2^n y)}{2^n} H(u)] = H(u)H(y) + H(y)H(u) \quad (2.14)$$

for all $u \in I_1(A_{sa})$ and all $y \in A$. By the assumption, we have

$$H(e) = \lim_n \frac{f(2^n e)}{2^n} \in U(B) \cap Z(B).$$

Similar to the proof of Theorem 2.1, it follows from (2.13) and (2.14) that $H = f$ on A. So H is continuous.

Putting $x = y = 0$ in (2.12), we have

$$\begin{aligned} \|H(u^*) - (H(u))^*\| &= \lim_n \|\frac{1}{2^n} f(2^n u^*) - \frac{1}{2^n}(f(2^n u))^*\| \\ &\le \lim_n \frac{1}{2^n}\phi(0, 0, 2^n u) \le \lim_n \frac{1}{2^n}\phi(0, 0, 2^n u) \\ &= 0 \end{aligned} \quad (2.15)$$

for all $u \in I_1(A_{sa})$. Since A is real rank zero, it is easy to show that $I_1(A_{sa})$ is dense in $\{x \in A_{sa} : \|x\| = 1\}$. Let $v \in \{x \in A_{sa} : \|x\| = 1\}$. Then there exists a sequence $\{z_n\}$ in $I_1(A_{sa})$ such that $\lim_n z_n = v$. Since H is continuous, it follows from (2.15) that

$$H(v^*) = H(\lim_n (z_n^*)) = \lim_n H(z_n^*) = \lim_n H(z_n)^* = H(\lim_n z_n)^* = H(v)^*. \quad (2.16)$$

Also, it follows from (2.14) that

$$\begin{aligned} H(vy + yv) &= H(\lim_n (z_n y + y z_n)) = \lim_n H(z_n y + y z_n) \\ &= \lim_n H(z_n)H(y) + \lim_n H(y)H(z_n) \\ &= H(\lim_n z_n)H(y) + H(y)H(\lim_n z_n) \\ &= H(v)H(y) + H(y)H(v) \end{aligned} \quad (2.17)$$

Now, let $x \in A$. Then we have $x = x_1 + ix_2$, where $x_1 := \frac{x+x^*}{2}$ and $x_2 := \frac{x-x^*}{2i}$ are self–adjoint.

First, consider the case that $x_1 \neq 0, x_2 \neq 0$. Since H is $\mathbb{C}-$linear, then it follows from (2.16) that

$$\begin{aligned} f(x^*) = H(x^*) = H((x_1 + ix_2)^*) &= H(\|x_1\|\frac{x_1^*}{\|x_1\|}) + H(i\|x_2\|\frac{x_2^*}{\|x_2\|}) \\ &= \|x_1\|H(\frac{x_1^*}{\|x_1\|}) - i\|x_2\|H(\frac{x_2^*}{\|x_2\|}) = \|x_1\|H(\frac{x_1}{\|x_1\|})^* - i\|x_2\|H(\frac{x_2}{\|x_2\|})^* \\ &= H(\|x_1\|\frac{x_1}{\|x_1\|})^* + H(i\|x_2\|\frac{x_2}{\|x_2\|})^* = [H(x_1) + H(ix_2)]^* \\ &= H(x)^* = f(x)^*. \end{aligned}$$

So, it follows from (2.17) that

$$f(xy + yx) = H(xy + yx) = H(x_1 y + ix_2 y + yx_1 + y(ix_2))$$

$$= H(\|x_1\|\frac{x_1}{\|x_1\|}y + y(\|x_1\|\frac{x_1}{\|x_1\|}) + H(i\|x_2\|\frac{x_2}{\|x_2\|}y + y(i\|x_2\|\frac{x_2}{\|x_2\|}))$$
$$= \|x_1\|H(\frac{x_1}{\|x_1\|}y + y\frac{x_1}{\|x_1\|}) + i\|x_2\|H(\frac{x_2}{\|x_2\|}y + y\frac{x_2}{\|x_2\|})$$
$$= \|x_1\|[H(\frac{x_1}{\|x_1\|})H(y) + H(y)H(\frac{x_1}{\|x_1\|})] + i\|x_2\|[H(\frac{x_2}{\|x_2\|})H(y) + H(y)H(\frac{x_2}{\|x_2\|})]$$
$$= [H(\|x_1\|\frac{x_1}{\|x_1\|}) + H(i\|x_2\|\frac{x_2}{\|x_2\|})]H(y) + H(y)[H(\|x_1\|\frac{x_1}{\|x_1\|}) + H(i\|x_2\|\frac{x_2}{\|x_2\|})]$$
$$= [H(x_1) + H(ix_2)]H(y) + H(y)[H(x_1) + H(ix_2)]$$
$$= H(x)H(y) + H(y)H(x) = f(x)f(y) + f(y)f(x)$$

for all $y \in A$.

Now, consider the case that $x_1 \neq 0, x_2 = 0$. Then it follows from (2.16) that

$$f(x^*) = H(x^*) = H((x_1)^*) = H(\|x_1\|\frac{x_1^*}{\|x_1\|}) = \|x_1\|H(\frac{x_1^*}{\|x_1\|}) = \|x_1\|H(\frac{x_1}{\|x_1\|})^*$$
$$= H(\|x_1\|\frac{x_1}{\|x_1\|})^* = H(x_1)^* = H(x)^* = f(x)^*.$$

Also, we have

$$f(xy + yx) = H(xy + yx) = H(x_1y + y(x_1)) = H(\|x_1\|\frac{x_1}{\|x_1\|}y + y(\|x_1\|\frac{x_1}{\|x_1\|}))$$
$$= \|x_1\|H(\frac{x_1}{\|x_1\|}y + y\frac{x_1}{\|x_1\|}) = \|x_1\|[H(\frac{x_1}{\|x_1\|})H(y) + H(y)H(\frac{x_1}{\|x_1\|})]$$
$$= H(\|x_1\|\frac{x_1}{\|x_1\|})H(y) + H(y)H(\|x_1\|\frac{x_1}{\|x_1\|}) = H(x_1)H(y) + H(y)H(x_1)$$
$$= H(x)H(y) + H(y)H(x) = f(x)f(y) + f(y)f(x)$$

for all $y \in A$.

Finally, consider the case that $x_1 = 0, x_2 \neq 0$. Then it follows from (2.16) that

$$f(x^*) = H(x^*) = H((ix_2)^*) = H(i\|x_2\|\frac{x_2^*}{\|x_2\|}) = -i\|x_2\|H(\frac{x_2^*}{\|x_2\|}) = -i\|x_2\|H(\frac{x_2}{\|x_2\|})^*$$
$$= H(i\|x_2\|\frac{x_2}{\|x_2\|})^* = H(ix_2)^* = H(x)^* = f(x)^*.$$

Similarly we can show that

$$f(xy + yx) = H(xy + yx) = H(x)H(y) + H(y)H(x) = f(x)f(y) + f(y)f(x)$$

for all $y \in A$. Hence, f is a Jordan $*-$homomorphism. □

Corollary 2.4. *Let $p \in (0,1), \theta \in [1,\infty)$ be real numbers. Let $f : A \to B$ be a mapping such that $f(0) = 0$ and*

$$f(2^nuy + 2^nyu) = f(2^nu)f(y) + f(y)f(2^nu)$$

for all $u \in I_1(A_{sa}), y \in A, n \in \mathbb{Z}_+$. Let

$$\|\mu f(\frac{x+y}{2}) + \mu f(\frac{x-y}{2}) - f(\mu x) + f(u^*) - f(u)^*\| \leq \theta(\|x\|^p + \|y\|^p + \|u\|^p)$$

for all $\mu \in \mathbb{T}$ and all $x, y \in A,\ u \in (I_1(A_{sa}) \cup \{0\})$.

If $\lim_n \frac{f(2^n e)}{2^n} \in U(B) \cap Z(B)$ *then the mapping* $f : A \to B$ *is a Jordan* $*-$*homomorphism.*

Proof. The conclusion follows from Theorem 2.3, by putting $\phi(x, y, u) := \theta(\|x\|^p + \|y\|^p + \|u\|^p)$ all $x, y, u \in A$ and $L = 2^{p-1}$. $\square$

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