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JORDAN $*-$HOMOMORPHISMS BETWEEN UNITAL C^*-ALGEBRAS: A FIXED POINT APPROACH

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Abstract. Let A, B be two unital C^*-algebras. By using fixed pint methods, we prove that:

a) Every almost unital almost linear mapping $h : A \longrightarrow B$ which satisfies $h(2^n uy + 2^n yu) = h(2^n u)h(y) + h(y)h(2^n u)$ for all $u \in U(A)$, all $y \in A$, and all $n = 0, 1, 2, ...$, is a Jordan homomorphism.

b) For a unital C^*-algebra A of real rank zero, every almost unital almost linear continuous mapping $h : A \longrightarrow B$ is a Jordan homomorphism when $h(2^n uy + 2^n yu) = h(2^n u)h(y) + h(y)h(2^n u)$ holds for all $u \in I_1(A_{sa})$, all $y \in A$, and all $n = 0, 1, 2, ...$.

Key Words and Phrases: Alternative fixed point, Jordan $*-$homomorphism.
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