

## A FIXED POINT THEOREM FOR GENERALIZED NONEXPANSIVE MULTIVALUED MAPPINGS

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**Abstract.** A mapping  $T$  on a normed space satisfies Suzuki condition provided that  $1/2\|x - Tx\| \leq \|x - y\|$  implies  $\|Tx - Ty\| \leq \|x - y\|$ . This notion which generalizes the concept of nonexpansiveness was recently introduced by Suzuki [Fixed point theorems and convergence theorems for some generalized nonexpansive mappings, *J. Math. Anal. Appl.* 340(2008) 1088–1095]. We shall first modify Suzuki condition to incorporate the class of multivalued mappings, and then prove a fixed point theorem for multivalued mappings satisfying the modified Suzuki condition. In this way, we generalize a theorem of T.C. Lim [A fixed point theorem for multivalued nonexpansive mappings in a uniformly convex Banach spaces, *Bull. Amer. Math. Soc.*, 80(1974) 1123–1126] proved for nonexpansive multivalued mappings to the class of generalized nonexpansive multivalued mappings in the above sense.

**Key Words and Phrases:** Fixed point, nonexpansive mapping, multivalued mapping, Suzuki condition, uniformly convex Banach space.

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