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EXISTENCE OF POSITIVE SOLUTIONS FOR SECOND ORDER DIFFERENTIAL EQUATION WITH FOUR POINT BOUNDARY CONDITIONS

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Abstract. We consider positive solutions to second-order four-point boundary value problem

$$\begin{cases} x''(t) + f(t, x, x') = 0, t \in (0, 1) \\ x(0) = \alpha x(\eta), x(1) = \beta x(\xi) \end{cases}$$

By using fixed point theorem, we present sufficient conditions which ensure the existence of three positive solutions to this problem. It's necessary to point out that it's the first time that positive solutions to this problem were established for the general case $\eta, \xi \in (0, 1)$. An examples is given to illustrate the main results.
Key Words and Phrases: Boundary value problem, positive solution, cone, fixed point.
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