

## ITERATIVE METHODS FOR VARIATIONAL INEQUALITIES, EQUILIBRIUM PROBLEMS, AND ASYMPTOTICALLY STRICT PSEUDOCONTRACTIVE MAPPINGS

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**Abstract.** In this paper, we introduce iterative algorithms for finding a common element of the set of fixed points for an asymptotically strict pseudocontractive mappings in the intermediate sense, the set of solutions of the variational inequalities for a family of  $\alpha$ -inverse-strongly monotone mappings and the set of solutions of a system of equilibrium problems in a Hilbert space. We establish some weak and strong convergence theorems of the sequences generated by our proposed algorithms. The strong convergence theorems are obtained via the hybrid method.

**Key Words and Phrases:** Asymptotically strict pseudocontractive mapping, equilibrium problem, inverse-strongly monotone mapping, iterative algorithm, projection.

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