

DOUBLE PROJECTION NEURAL NETWORK MODEL FOR SOLVING PSEUDOMONOTONE VARIATIONAL INEQUALITIES

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Abstract. In this paper we propose a novel double projection recurrent neural network model for solving pseudomonotone variational inequalities based on a technique of updating the state variable and fixed point theorem. This model is stable in the sense of Lyapunov and globally convergent for problems that satisfy Lipschitz continuity and pseudomonotonicity conditions. The global exponential stability of the model under the assumptions of strong pseudomonotonicity and Lipschitz continuity is proved. Numerical simulation to various types of variational inequalities is given to show the applied significance of the results

Key Words and Phrases: Pseudomonotone variational inequalities, Double projection neural network, Fixed point theory, Lyapunov stability, Globally convergence.

2010 Mathematics Subject Classification: 58E35, 47H10, 68T20.

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Received: August 1, 2009; Accepted: November 5, 2009.