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QUASI-FIXED POLYNOMIAL FOR VECTOR-VALUED POLYNOMIAL FUNCTIONS ON $\mathbb{R}^n \times \mathbb{R}$

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Abstract. Let $F : \mathbb{R}^n \times \mathbb{R} \longrightarrow \mathbb{R}^k$ be a vector-valued polynomial function:

$$F(\overline{x}, y) = \big(F_1, F_2, ..., F_k\big)(\overline{x}, y), \qquad \overline{x} \in \mathbb{R}^n, \quad y \in \mathbb{R}.$$

Each component F_i of F is a real-valued polynomial function, the degree of y of F_i is $\deg_y F_i = s_i$, and is represented by:

$$F_i(\overline{x}, y) = \sum_{j=0}^{s_i} f_{i,j}(\overline{x}) y^j, \quad i = 1, 2, \ldots, k,$$

where $f_{i,j}(\overline{x}) \in \mathbb{R}[\overline{x}]$.

In this paper, for each F_i, we give an irreducible polynomial $p_i^{m_i}(\overline{x})$ of m_i-power and consider a real-valued quasi-fixed point problem as the form:

$$F_i(\overline{x}, y) = a_i p_i^{m_i}(\overline{x}), \quad i = 1, 2, \cdots, k.$$

We aim to find a polynomial function $y = y(\overline{x})$, $\overline{x} \in \mathbb{R}^n$ to satisfy the following vector-valued polynomial equation:

$$(\divideontimes) \quad F(\overline{x}, y(\overline{x})) = \big(a_1 p_1^{m_1}(\overline{x}), a_2 p_2^{m_2}(\overline{x}), \cdots, a_n p_k^{m_k}(\overline{x})\big),$$

where $(a_1, a_2, \ldots, a_k) \in \mathbb{R}^k$ is a constant vector depending on the solution $y(\overline{x})$. We will investigate the solution sets of $(\divideontimes)$ and containing either (i) of finitely many or (ii) of infinitely many quasi-fixed (point) solutions. In case of (i), the number of solutions do not exceed

$$\max_{1 \le i \le k} \{s_i + 2\}.$$

While the case (ii), all solutions are represented as the form

$$\{-f_{s_i - 1}(\overline{x})/s_i f_{s_i}(\overline{x}) + \lambda p^t(\overline{x}) : \text{ for all } \lambda \in \mathbb{R}\}$$

where $t \le m_i/s_i$ for any i, $1 \le i \le k$.
Key Words and Phrases: Quasi-fixed point (solution), quasi-fixed (constant) vector.
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