

WEAK CONVERGENCE THEOREMS FOR EQUILIBRIUM PROBLEMS WITH NONLINEAR OPERATORS IN HILBERT SPACES

S. DHOMPONGSA*, W. TAKAHASHI** AND H. YINGTAWESITTIKUL***

*Department of Mathematics, Faculty of Science
Chiang Mai University, Chiang Mai 50200, Thailand.
E-mail: sompong@chiangmai.ac.th

**Department of Mathematical and Computing Sciences
Tokyo Institute of Technology, Ohokayama, Meguro-ku
Tokyo 152-8552, Japan.
E-mail: wataru@is.titech.ac.jp

***Department of Mathematics, Faculty of Science
Chiang Mai University, Chiang Mai 50200, Thailand.
E-mail: g4825119@cm.edu

Abstract. In this paper, we introduce an iterative sequence for finding a common element of the set of fixed points of a nonspreading mapping, the set of solutions of an equilibrium problem and the set of solutions of the variational inequality problem for a monotone and Lipschitz-continuous mapping. We show that the sequence converges weakly to a common element of the above three sets.

Key Words and Phrases: Nonspreading mappings, monotone, Lipschitz-continuous mappings, variational inequalities, fixed points.

2010 Mathematics Subject Classification: 47H05, 47H09, 47H20.

Acknowledgement. The authors would like to thank the Thailand Research Fund (grant BRG4780016) and the Development and Promotion of Science and Technology Talent Project (DPST) for their support.

REFERENCES

- [1] E. Blum and W. Oettli, *From optimization and variational inequalities to equilibrium problems*, Math. Student, **63**(1994), 123-145.
- [2] P.L. Combettes and A. Hirstoaga, *Equilibrium programming in Hilbert spaces*, J. Nonlinear Convex Anal., **6**(2005), 117-136.
- [3] F. Kosaka and W. Takahashi, *Existence and approximation of fixed points of firmly nonexpansive-type mappings in Banach spaces*, SIAM. J. Optim., **19**(2008), 824-835.
- [4] F. Kosaka and W. Takahashi, *Fixed point theorems for a class of nonlinear mappings related to maximal monotone operators in Banach spaces*, Arch. Math. (Basel), **91**(2008), 166-177.
- [5] N. Nadezhkina and W. Takahashi, *Weak convergence theorem by an extragradient Method for Nonexpansive mappings and Monotone Mappings*, J. Optim. Theory Appl., **128**(2006), 191-201.

*Corresponding author.

- [6] Z. Opial, *Weak convergence of the sequence of successive approximations for nonexpansive mappings*, Bull. Amer. Math. Soc., **73**(1967), 591-597.
- [7] R.T. Rockafellar, *On the maximality of sums of nonlinear monotone operators*, Trans. Amer. Math. Soc., **149**(1970), 75-88.
- [8] J. Schu, *Weak and strong convergence to fixed points of asymptotically nonexpansive mappings*, Bull. Austral. Math. Soc., **43**(1991), 153-159.
- [9] A. Tada and W. Takahashi, *Strong convergence theorem for an equilibrium problem and a nonexpansive mapping*, in: Nonlinear Analysis and Convex Analysis, (W. Takahashi and T. Tanaka-Eds.), Yokohama Publishers, Yokohama, 2007, 609-617.
- [10] A. Tada and W. Takahashi, *Weak and strong convergence theorems for a nonexpansive mapping and equilibrium problem*, J. Optim. Theory Appl., **133**(2007), 359-370.
- [11] S. Takahashi and W. Takahashi, *Viscosity approximation methods for equilibrium problems and fixed point problems in Hilbert spaces*, J. Math. Anal. Appl., **331**(2007), 506-515.
- [12] W. Takahashi, *Introduction to Nonlinear and Convex Analysis*, Yokohama Publishers, Yokohama, 2009.
- [13] W. Takahashi and M. Toyoda, *Weak convergence theorems for nonexpansive mappings and monotone mappings*, J. Optim. Theory Appl., **118**(2003), 417-428.

Received: March 4, 2010; Accepted: June 11, 2010.