ITERATIVE METHODS FOR GENERALIZED EQUILIBRIUM PROBLEMS, SYSTEMS OF GENERAL GENERALIZED EQUILIBRIUM PROBLEMS AND FIXED POINT PROBLEMS FOR NONEXPANSIVE MAPPINGS IN HILBERT SPACES

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Abstract. In this paper, we introduce a system of general generalized equilibrium problems and propose an iterative scheme for finding the approximate solutions of a generalized equilibrium problem, a system of general generalized equilibrium problems and a fixed point problem of a nonexpansive mapping in a Hilbert space. We establish a strong convergence theorem for a sequence generated by our proposed iterative scheme to a common solution of these three problems. Utilizing this result, we prove three new strong convergence theorems for sequences generated by iterative schemes for fixed point problems, variational inequalities, equilibrium problems and systems of general generalized equilibrium problems.
Key Words and Phrases: Generalized equilibrium problems, system of variational inequalities, fixed point problems, inverse-strongly monotone mappings, iterative methods, strong convergence.
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