Fixed Point Theory, 12(2011), No. 2, 285-292
http://www.math.ubbcluj.ro/~nodeacj/sfptcj.html

COMMON FIXED POINTS OF BANACH OPERATOR PAIR ON FUZZY NORMED SPACES

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Abstract. We prove the existence of common fixed points of noncommuting mappings on fuzzy normed spaces.
Key Words and Phrases: Fixed point, fuzzy normed space, fuzzy nonexpansive mapping, $R-$subweakly commuting mappings, Banach operator pair.
2010 Mathematics Subject Classification: 47H10, 47H09, 54H25, 46S40.

1. Introduction and preliminaries

The evolution of fuzzy mathematics commenced with an introduction of the notion of fuzzy sets by Zadeh [21] in 1965, as a new way to represent vagueness in every day life. The concept of a fuzzy metric space was introduced and generalized in many ways ([8], [14]). Moreover George and Veeramani ([11], [12]) modified the concept of a fuzzy metric space introduced by Kramosil and Michalek [13]. They obtained a Hausdorff topology for this kind of fuzzy metric spaces which has applications in quantum particle physics, particularly in connection with both string and ϵ^{∞} theory (see [9] and references mentioned therein). Many authors have proved fixed point and common fixed point theorems in fuzzy metric spaces ([1],[5], [15], [17]). Fixed point theorems in these spaces have applications to control theory, system theory and optimization problems. The study of fixed points theory in fuzzy normed spaces is a very recent development ([2], [3], [4] and [6]). Recently Chen and Li [7] introduced the class of Banach operator pairs, as a new class of noncommuting maps and proved some common fixed point results on normed spaces. On the other hand, Beg et al. [6] obtained common fixed point of uniformly $R-$ subweakly commuting mappings in fuzzy Banach spaces. In this paper, we obtain common fixed points for Banach operator pair on fuzzy normed spaces. The main feature of our results is that we relax the condition of linearity of one of the mapping involved therein, which is key assumption in the results of [6]. Our results extend, generalize and unify various known results in the existing literature.

For the sake of convenience, we first give following definitions and known results.

Definition 1.1. ([6]) Let U be a linear space over the field of real numbers $\mathbb{R}$. A fuzzy subset N of $U \times \mathbb{R}$ is called a *fuzzy norm* on U if and only if for all $x, y \in U$ and $c \in \mathbb{R}$, the following conditions are satisfied:

(N1) For all $t \in \mathbb{R}$ with $t \leq 0$, $N(x,t) = 0$;
(N2) For all $t \in \mathbb{R}$ with $t > 0$, $N(x,t) = 1$ if and only if $x = 0$;
(N3) For all $t \in \mathbb{R}$ with $t > 0$, $N(cx,t) = N(x, \frac{t}{|c|})$ if $c \neq 0$;
(N4) For all $t, s \in \mathbb{R}$, $N(x+y, t+s) \geq \min\{N(x,t), N(y,s)\}$;
(N5) $N(x, .) : (0, \infty) \longrightarrow [0,1]$ is continuous, and $\lim\limits_{t\to\infty} N(x,t) = 1$.

The pair (U, N) is called a *fuzzy normed space.*

Definition 1.2. Let (U, N) be a fuzzy normed space. We define an *open ball* $B(x,r,t)$ and a *closed ball* $B[x,r,t]$ with a center $x \in U$ and a radius $0 < r < 1$, $t > 0$ as follows:

$$\begin{aligned} B(x,r,t) &= \{y \in X : N(x-y,t) > 1-r\}, \\ B[x,r,t] &= \{y \in X : N(x-y,t) \geq 1-r\}. \end{aligned}$$

Definition 1.3. Let (U, N) be a fuzzy normed space. A sequence $\{x_n\}$ in U is said to be *Cauchy* if $\lim\limits_{n\to\infty} N(x_{n+p} - x_n, t) = 1$ for all $t > 0$ and $p = 1, 2, 3, ...$

Definition 1.4. Let (U, N) be a fuzzy normed space. A sequence $\{x_n\}$ in U is said to be *convergent* if there exists an $x \in U$ such that $\lim\limits_{n\to\infty} N(x_n - x, t) = 1$ for all $t > 0$. In this case x is called *limit of a sequence* $\{x_n\}$.

A fuzzy normed space U is said to be *complete* if every Cauchy sequence in U is convergent in U. A complete fuzzy normed space U is called a *fuzzy Banach space.*

Definition 1.5. Let (U, N) be a fuzzy normed space and $f : U \to U$. A mapping $T : U \to U$ is called *fuzzy $f-$nonexpansive* if for all $x, y \in U$,

$$N(Tx - Ty, t) \geq N(fx - fy, t),$$

for all $t \in \mathbb{R}$.

If we put $f = I$ (identity map) in Definition 1.5, we obtain definition 4.5 of [3].

Definition 1.6. Let (U, N) be a fuzzy normed space and $f : U \to U$. A mapping $T : U \to U$ is called *fuzzy asymptotically $f-$nonexpansive* if there exists a sequence $\{k_n\}$ of real numbers in $[1, \infty)$ with $\lim\limits_{n\to\infty} k_n = 1$ such that

$$N(T^n x - T^n y, t) \geq N(fx - fy, \frac{t}{k_n}),$$

for $x, y \in U$, $t \in \mathbb{R}$.

Definition 1.7. Let (U, N) be a fuzzy normed space and T, $S : U \to U$. A point $x \in U$ is called:

(1) *a fixed point of* T if $T(x) = x$;
(2) *a coincidence point of the pair* $\{T, S\}$ if $Tx = Sx$;
(3) *a common fixed point of the pair* $\{T, S\}$ if $x = Tx = Sx$.

$F(T), C(T,S)$ and $F(T,S)$ denote set of all fixed points of T, the set of all coincidence points of the pair $\{T, S\}$, and the set of all common fixed points of the pair $\{T, S\}$, respectively. For any x, u in U we denote

$$[x, u] = \{tx + (1-t)u : 0 \leq t \leq 1\}.$$

Definition 1.8. Let C be a nonempty subset of a fuzzy normed space (U, N) and S, T be self-mappings of C. Then T is said to be
(1) an $S-$ *contraction* if there exists $k \in (0,1)$ such that

$$N(Tx - Ty, t) \geq N(Sx - Sy, \frac{t}{k})$$

for all $x, y \in C$;
(2) a *uniformly asymptotically regular* on C if, for each $0 < \varepsilon < 1$, there exists $n(\varepsilon) = n_0$ such that

$$N(T^n x - T^{n+1}x, t) > 1 - \varepsilon,$$

for all $n > n_0$ and $x \in C$.
(3) *commuting* on C if $TSx = STx$ for all $x \in C$;
(4) *weakly compatible* if $TSx = STx$ for all $x \in C(T, S)$;
(5) $R-$*weakly commuting* on C if there exists a real number $R > 0$ such that $N(TSx - STx, t) \geq N(Tx - Sx, \frac{t}{R})$ for all $x \in C$, $t \in \mathbb{R}$.
(6) C_q-*commuting* if $STx = TSx$ for all $x \in C_q(S,T)$, where $C_q(S,T) = \cup\{C(S, T_k) : 0 \leq k \leq 1\}$ and $T_k x = (1-k)q + kTx$.

Note that there exists a subset D of X on which the pair (S, T) is $R-$weakly commuting. However, the pair is not $R-$weakly commuting on the entire space [6].
Let $q \in C$. The set C is called $q-$*starshaped* if $[x, q] \subseteq C$ for all $x \in C$. Note that C is *convex* if C is $q-$starshaped for every $q \in C$.

Definition 1.9. ([6]) Let C be a nonempty convex subset of a fuzzy normed space (U, N). Let $S, T : C \longrightarrow C$ be two mappings. Then S and T are said to be $R-$ *subweakly commuting* on C if there exists a real number $R > 0$ and a $u \in C$ such that

$$N(TSx - STx, t) \geq dist(Sx, [Tx, u], \frac{t}{R})$$

for all $x \in C$, $t \in \mathbb{R}$ where

$$dist(Sx, [Tx, u], t) = \sup\{N(Sx - z, t) : z \in [Tx, u]\}.$$

Definition 1.10. Let C be a nonempty convex subset of a fuzzy normed space (U, N). Let S, $T : C \longrightarrow C$ be two mappings. Then S and T are said to be *uniformly $R-$ subweakly commuting* on C if there exists a real number $R > 0$ and a u in C such that

$$N(T^n Sx - ST^n x, t) \geq dist(Sx, [T^n x, u], \frac{t}{R})$$

for all $x \in C, t \in \mathbb{R}$ and $n \in \mathbb{N}$.

C_q-commuting maps are weakly compatible but not conversely in general and uniformly $R-$subweakly commuting maps are $R-$subweakly commuting and $R-$subweakly commuting maps are C_q-commuting but the converse does not hold in general.

Definition 1.11. The ordered pair (T, S) of two self maps of a a fuzzy normed space (U, N) is called a *Banach operator pair* if the set $F(S)$ is T invariant, namely $T(F(S)) \subseteq F(S)$.
Obviously in general any commuting pair (T, S) is a Banach operator pair but not conversely.
Next we give an example of noncommuting Banach operator pair. It also illustrates the significance of order in which mappings S and T appear in the pair. However for selfmaps S and T with $F(T) = F(S)$, (T, S) and (S, T) are Banach operator pairs.
Example 1.12. Take $U = \mathbb{R}$ and N the fuzzy set on $U \times \mathbb{R}$ defined by

$$N(x,t) = \begin{cases} 0, & t \leq 0 \\ \dfrac{t}{t+|x|}, & t > 0 \end{cases}$$

Let $C = [0, 1]$ and $S, T : C \to C$ given by

$$T(x) = \begin{cases} 1, & x = 0 \\ \dfrac{1}{2}, & x \in (0,1] \end{cases}$$

and

$$S(x) = \begin{cases} 0, & x = [0, \frac{1}{2}) \\ \dfrac{1}{2}, & x \in [\frac{1}{2}, 1). \end{cases}$$

Here $F(T) = \{\frac{1}{2}\}$, and $F(S) = \{0, \frac{1}{2}\}$. Note that $F(T)$ is S invariant but $F(S)$ is not T invariant. Also, $ST(0) = \frac{1}{2} \neq TS(0) = 1$ implies that T and S are not commuting on C. However it is C_0- commuting and C_1-commuting.
Following is an example of Banach operator pair which is not uniformly $R-$subweakly commuting [16].

Example 1.13. Take $U = \mathbb{R}$ and N be the fuzzy set on $U \times \mathbb{R}$ defined by

$$N(x,t) = \begin{cases} 0, & t \leq 0 \\ \dfrac{t}{t+|x|}, & t > 0. \end{cases}$$

Let $C = [1, \infty)$. Let $T(x) = x^2$ and $S(x) = 2x - 1$, for all $x \in C$. Let $q = 1$. Then C is convex with $q \in F(S)$, $F(S) = \{1\}$ and $C_q(S, T) = [1, \infty)$. Note that the pair (T, S) is Banach operator pair but T and S are not C_q-commuting maps. Hence T and S are not $R-$subweakly and uniformly $R-$subweakly commuting maps.

2. Common fixed points

In this section, common fixed points for Banach operator pair on fuzzy normed spaces are obtained. First, we prove the following theorem which is needed to extend recent common fixed point results of [6] to a new class of noncommuting mappings.

Theorem 2.1. Let C be a nonempty closed subset of a fuzzy normed space (U, N). Let $S, T : C \longrightarrow C$ be weakly compatible mappings. Assume that $clT(C) \subseteq S(C)$, $clT(C)$ is complete and T is an $S-$ contraction on C, then $F(S) \cap F(T)$ is a singleton.
Proof. Let $x_0 \in C$. Since $T(C) \subset S(C)$, we can define a sequence $\{x_n\}$ in C by $Sx_n = Tx_{n-1}$ for $n \geq 1$. Then

$$\begin{aligned} N(Sx_{n+1} - Sx_n, t) &= N(Tx_n - Tx_{n-1}, t) \\ &\geq N(Sx_n - Sx_{n-1}, t/k), \end{aligned}$$

for some $k \in (0, 1)$. Sedghi and Shobe [19] implies that $\{Sx_n\}$ is a Cauchy sequence in C and $\{Tx_n\}$ is also a Cauchy sequence in $clT(C)$. Thus, there exists y in $clT(C)$ such that $Tx_n \longrightarrow y$. Consequently, $Sx_n \longrightarrow y$. Then since $clT(C) \subseteq S(C)$, there exists a point u in C such that $y = Su$. Further, we have

$$N(Tx_n - Tu, t) \geq N(Sx_n - Su, t/k).$$

Taking limit as $n \longrightarrow \infty$, above inequality yields $y = Tu$. Thus u is a coincidence point of T and S. So by weak compatibility of $\{T, S\}$, it follows that $STu = TSu$ and so $Sy = Ty$. Next we show that y is a common fixed point of S and T. Since,

$$N(Tx_n - Ty, t) \geq N(Sx_n - Sy, t/k).$$

Taking limit as $n \longrightarrow \infty$ yields $Ty = Sy = y$. The uniqueness follows from the contraction condition. □
Example 2.2. Take $U = \mathbb{R}$ and N the fuzzy set on $U \times \mathbb{R}$ defined by

$$N(x, t) = \begin{cases} 0, & t \leq 0 \\ \dfrac{t}{t + |x|}, & t > 0 \end{cases}$$

Let $C = [\frac{3}{4}, \infty)$ and $S, T : C \to C$ given by $Tx = \frac{2}{3}x + 1$ and $Sx = 2x$. It may be verified that

$$N(Tx - Ty, t) \geq N(Sx - Sy, \frac{t}{k})$$

holds for all $x, y \in C$, where $k = \dfrac{2}{3} < 1$. Moreover S and T have a coincidence point in C.
In above example, S and T do not commute at the coincidence point $\frac{3}{4}$, and therefore are not weakly compatible. And S and T do not have a common fixed point. Thus this example explains the role of weak compatibility in our above result.
Corollary 2.3. Let C be a nonempty closed subset of a fuzzy normed space (U, N). Let $S, T : C \longrightarrow C$ be commuting mappings. Assume that $clT(C) \subseteq S(C)$, $clT(C)$ is complete and T is an $S-$ contraction on C, then S and T have a common fixed point.
Corollary 2.4. Let C be a nonempty closed subset of a fuzzy normed space (U, N). Let $T : C \longrightarrow C$ be a contraction mapping on C. Assume that $clT(C)$ is complete, then T has a unique fixed point.
Lemma 2.5. Let C be a nonempty closed subset of a fuzzy normed space (U, N). Let (T, S) be a Banach operator pair on C. Assume that $clT(C)$ is complete and T

is an $S-$ contraction on C. If S is continuous and $F(S)$ is nonempty, then S and T have a common fixed point.

Proof. By our assumptions, $T(F(S)) \subseteq F(S)$ and $F(S)$ is nonempty and closed. Moreover, $cl(T(F(S)))$ being subset of $cl(T(C))$ is complete. Further for all $x,\ y \in F(S)$, we have

$$N(Tx - Ty, t) \geq N(Sx - Sy, t/k) = N(x - y, t/k).$$

Hence T is a contraction on $F(S)$ and $cl(T(F(S))) \subseteq cl(F(S)) = F(S)$. By Theorem 2.1, T has a unique fixed point z in $F(S)$ and consequently $F(S) \cap F(T)$ is singleton. □

Now we present an example in the support of above lemma which is crucial for the proof of Theorem 2.7.

Example 2.6. Take $U = \mathbb{R}$ and N the fuzzy set on $U \times \mathbb{R}$ defined by

$$N(x,t) = \begin{cases} 0, & t \leq 0 \\ \dfrac{t}{t + |x|}, & t > 0 \end{cases}$$

Let $C = [1, \infty)$ and $S, T : C \to C$ given by

$$Tx = 2x - 1$$

and

$$Sx = x^3$$

Note that for all $x, y \in C$

$$N(Tx - Ty, t) \geq N(Sx - Sy, \frac{t}{k})$$

holds for $k = \dfrac{3}{4}$. All conditions of lemma 2.6 are satisfied. Moreover 1 is the unique common fixed point of S and T.

Let C be a nonempty $q-$starshaped subset of a fuzzy normed space (U, N) and $T : C \to C$. For $n \in \mathbb{N}$, define a mapping T_n on C by

$$T_n x = \mu_n T^n x + (1 - \mu_n) q,\ x \in C,$$

where $\mu_n = \frac{\lambda_n}{k_n}$, $\{\lambda_n\}$ is a sequence of real numbers in $(0,1)$ such that $\lim\limits_{n \to \infty} \lambda_n = 1$.

Theorem 2.7. Let S, T be two self-mappings of a nonempty $q-$ starshaped subset C of a fuzzy normed space (U, N). Assume that S is continuous and $F(S)$ is $q-$starshaped with respect to some q in $F(S)$, (T, S) is a Banach operator pair on C and satisfy for each $n \geq 1$

$$N(T^n x - T^n y, t) \geq N(Sx - Sy, \frac{t}{k_n})$$

for all $x, y \in C$, $\{k_n\}$ is a sequence of real numbers in $[1, \infty)$ such that $\lim\limits_{n \to \infty} k_n = 1$. Then for each $n \geq 1$, T_n and S have exactly one common fixed point x_n in C provided $cl(T_n(C))$ is complete for each n.

Proof. By definition,

$$T_n x = \mu_n T^n x + (1 - \mu_n) q.$$

As (T,S) is a Banach operator pair, for each $n \geq 1$, $T^n(F(S)) \subseteq F(S)$ and $F(S)$ is nonempty and closed. Since $F(S)$ is $q-$starshaped and $T^n x \in F(S)$, thus for each $x \in F(S)$,

$$T_n x = (1-\mu_n)q + \mu_n T^n x \in F(S).$$

Thus (T_n, S) is a Banach operator pair for each n. Also, for all $x, y \in C$, we have

$$\begin{aligned} N(T_n x - T_n y, t) &= N(\mu_n(T^n x - T^n y), t) \\ &= N(T^n x - T^n y, \frac{t}{\mu_n}) \\ &\geq N(Sx - Sy, \frac{t}{\mu_n k_n}). \end{aligned}$$

It follows that T_n is $S-$ contraction. Also, T_n is a self-mapping of C. By Lemma 2.5, for each $n \geq 1$, there exists a unique $x_n \in C$ such that $x_n = Sx_n = T_n x_n$. □

Theorem 2.8. Let S, T be self-mappings of nonempty $q-$ starshaped subset C of fuzzy normed space (U, N). Assume that (T,S) is a Banach operator pair on C, S is continuous and $F(S)$ is $q-$ starshaped with respect to some $q \in F(S)$. Suppose T is uniformly asymptotically regular and asymptotically $S-$nonexpansive, then S and T have a common fixed point in C provide $cl(T(C))$ is compact and T is continuous.

Proof. Notice that compactness of $cl(T(C))$ implies that $clT_n(C)$ is compact and thus complete. From Theorem 2.7, for each $n \geq 1$, there exists $x_n \in C$ such that

$$x_n = Sx_n = (1-\mu_n)q + \mu_n T^n x_n.$$

Also

$$N(x_n - T^n x_n, t) = N(1-\mu_n)(T^n x_n - u), t).$$

Since $T(C)$ is bounded, $N(x_n - T^n x_n, t) \longrightarrow 1$ as $n \to \infty$. Since (T,S) is a Banach operator pair and $Sx_n = x_n$, so $ST^n x_n = T^n S x_n = T^n x_n$, thus we have

$$\begin{aligned} & N(x_n - Tx_n, t) \\ \geq\ & \min\{N(x_n - T^n x_n, \frac{t}{3}), N(T^n x_n - T^{n+1} x_n, \frac{t}{3}), N(T^{n+1} x_n - Tx_n, \frac{t}{3})\} \\ \geq\ & \min\{N(x_n - T^n x_n, \frac{t}{3}), N(T^n x_n - T^{n+1} x_n, \frac{t}{3}), N(ST^n x_n - Sx_n, \frac{t}{3k})\} \\ =\ & \min\{N(x_n - T^n x_n, \frac{t}{3}), N(T^n x_n - T^{n+1} x_n, \frac{t}{3}), N(T^n x_n - x_n, \frac{t}{3k})\}. \end{aligned}$$

Since T is uniformly asymptotically regular, we have $N(Tx_n - x_n, t) \longrightarrow 1$ as $n \longrightarrow \infty$. Since $cl(T(C))$ is compact, there exists a subsequence $\{Tx_m\}$ of $\{Tx_n\}$ such that $N(Tx_m - x_m, t) \longrightarrow 1$ as $m \longrightarrow \infty$. By the continuity of S and T and the fact $N(Tx_m - x_m, t) \longrightarrow 1$ as $m \longrightarrow \infty$, we have $y \in F(T) \cap F(S)$. Thus $F(T) \cap F(S) \neq \phi$.
□

Acknowledgements. The present version of the paper owes much to the precise and kind remarks of the learned referee.We also acknowledge with thanks Higher Education Commission of Pakistan research grant: 20-918/R&D/07.

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Received: December 10, 2008; Accepted: June 4, 2009.