

FUNCTION SPACES WITH THE MATKOWSKI PROPERTY AND DEGENERACY PHENOMENA FOR COMPOSITION OPERATORS

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Abstract. We give a condition, both necessary and sufficient, on a function $f: \mathbb{R} \rightarrow \mathbb{R}$, under which the nonlinear composition operator F defined by $Fx(t) = f(x(t))$ satisfies a local Lipschitz condition in the norm of the function spaces $C^1([a, b])$, $C^{0,\alpha}([a, b])$, $Lip([a, b])$, and $BV([a, b])$. In contrast to global Lipschitz conditions, this does not lead to a strong degeneracy of the generating function f , which is important to apply fixed point theorems.

Key Words and Phrases: Function spaces, nonlinear composition operator, global Lipschitz condition, local Lipschitz condition, Matkowski property, weak Matkowski property.

2010 Mathematics Subject Classification: 47H30, 26A16, 26A24, 26A45, 26A46, 47H09, 47H10.

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¹The research is supported by the Academy of Sciences of the Czech Republic under the Grant IAA100190805 of the GAAV and the Institutional Research Plan AV0Z10190503.

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Received: June 6, 2010; Accepted: January 11, 2011.