

ALGORITHMS FOR APPROXIMATING MINIMIZATION PROBLEMS

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Abstract. In this paper, we study the following minimization problem

$$\min_{x \in F(S) \cap \Omega} \frac{\mu}{2} \langle Bx, x \rangle + \frac{1}{2} \|x\|^2,$$

where B is a bounded linear operator, $\mu \geq 0$ is some constant, $F(T)$ is the set of fixed points of nonexpansive mapping S and Ω is the solution set of an equilibrium problem. This paper introduces two new algorithms (one implicit and one explicit) that can be used to find the solution of the above minimization problem.

Key Words and Phrases: Nonexpansive mapping, monotone mapping, fixed point, equilibrium problem, minimization problem.

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