Fixed Point Theory, 12(2011), No. 1, 187-204
http://www.math.ubbcluj.ro/~nodeacj/sfptcj.html

APPLICATIONS OF THE S-ITERATION PROCESS TO CONSTRAINED MINIMIZATION PROBLEMS AND SPLIT FEASIBILITY PROBLEMS

D. R. SAHU

Department of Mathematics, Banaras Hindu University
Varanasi-221005, India
E-mail: drsahudr@gmail.com

Abstract. In this paper the S-iteration process introduced by Agarwal, O'Regan and Sahu [Iterative construction of fixed points of nearly asymptotically nonexpansive mappings, J. Nonlinear Convex Anal., 8 (2007), 61-79] is further analyzed for contraction and nonexpansive mappings. It is shown, theoretically as well as numerically, that the S-iteration process is faster than the Picard and KM-iteration processes for contraction operators. We also propose a new iterative algorithm and prove a strong convergence theorem for computing fixed points of nonexpansive operators in a Banach space. Our results are applied for finding solutions of constrained minimization problems and split feasibility problems. Our iteration methods are of independent interest.
Key Words and Phrases: Accretive operator, nonexpansive mapping, sunny nonexpansive retraction, fixed point iterative algorithm, normal S-iteration process, rate of convergence of iterative algorithm, constrained optimization problem, split feasibility problem.
2010 Mathematics Subject Classification: 47H09, 47H10.

Acknowledgements. Author thanks the referee for his/her valuable suggestions for the manuscript.

References

[1] R.P. Agarwal, D. O'Regan, D.R. Sahu, *Fixed Point Theory for Lipschitzian-type Mappings with Applications*, Series Topological Fixed Point Theory and Its Applications, Springer, New York, 2009.
[2] R.P. Agarwal, D. O'Regan, D.R. Sahu, *Iterative construction of fixed points of nearly asymptotically nonexpansive mappings*, J. Nonlinear Convex Anal., **8**(2007), no. 1, 61-79.
[3] J.P. Aubin, A. Cellina, *Diffierential Inclusions*, Springer, Berlin, 1984.
[4] J B. Baillon, G. Haddad, *Quelques proprietes des operateurs angle-bornes et cycliquement monotones*, Israel J. Math., **26**(1977), 137-150.
[5] V. Berinde, *Picard iteration converges faster than Mann iteration for a class of quasicontractive operators*, Fixed Point Theory and Applications, **2004**(2004), 97-105.
[6] F.E. Browder, *Convergence of approximants to fixed points of nonexpansive nonlinear mappings in Banach spaces*, Arch. Rational Mech. Anal., **24**(1967), 82-90.
[7] C.L. Byrne, *A unified treatment of some iterative algorithms insignal processing and image reconstruction*, Inverse Problems, **20**(2004), 103-120.
[8] C.E. Chidume, C.O. Chidume, *Iterative approximation of fixed points of nonexpansive mappings*, J. Math. Anal. Appl., **318**(2006), no. 1, 288-295.

[9] Y.J. Cho, S.M. Kang, H.Y. Zhou, *Some control condition on iterative methods*, Commun. Appl. Nonlinear Anal., **12**(2005), no. 2, 27-34.
[10] P. Combettes, *Fejer monotonicity in convex optimization,* Encyclopedia of Optimization (C.A. Floudas and P.M. Pardalos, Eds.), Boston, MA, Kluwer, 2000.
[11] B. Halpern, *Fixed points of nonexpanding maps*, Bull. Amer. Math. Soc., **73**(1967), 957-961.
[12] C.I. Podilchuk, R.J. Mammone, *Image recovery by convex projections using a least squares constraint*, J. Opt. Soc. Am. A, **7**(1990), 517-521.
[13] D. Kinderlehrer, G. Stampacchia, *An Introduction to Variational Inequalities and their Applications*, Academic Press, New York, 1980.
[14] P.L. Lions, *Approximation de points fixes de contractions*, C.R. Acad. Sci. Paris Ser. AB, **284**(1977), 1357-1359.
[15] J.L. Lions, G. Stampacchia, *Variational inequalities*, Comm. Pure Appl. Math., **20**(1967), 493-517.
[16] S. Reich, *Strong convergence theorems for resolvents of operators in Banach spaces,* J. Math. Anal. Appl., **75**(1980), 287-292.
[17] S. Reich, *Weak convergence theorems for nonexpansive mappings in Banach spaces*, J. Math. Anal. Appl., **67**(1979), 274276.
[18] B.E. Rhoades, *Comments on two fixed point iteration methods*, J. Math. Anal. Appl., **56**(1976), no. 3, 741-750.
[19] D.R. Sahu, Zeqing Liu and Shin Min Kang, *Iterative approaches to common fixed points of asymptotically nonexpansive mappings*, Rocky Mountain J. Math., **39**(2009), 281-304.
[20] T. Suzuki, *Strong convergence of approximated sequences for nonexpansive mappings in Banach spaces*, Proc. Amer. Math. Soc., **135**(2007), 99-106.
[21] W. Takahashi, *Nonlinear Functional Analysis*, Yokohama Publishers, Yokohama, 2000.
[22] W. Takahashi, Y. Ueda, *On Reich's strong convergence theorems for resolvents of accretive operators*, J. Math. Anal. Appl., **104**(1984), 546-553.
[23] K.K. Tan, H.K. Xu, *Approximating fixed points of nonexpansive mappings by the Ishikawa iteration process,* J. Math. Anal. Appl., **178**(1993), 301-308.
[24] N. Shioji, W. Takahashi, *Strong convergence of approximated sequences for nonexpansive mappings in Banach spaces*, Proc. Amer. Math. Soc., **125**(1997), 3641-3645.
[25] R. Wittmann, *Approximation of fixed points of nonexpansive mappings*, Arch. Math., **58**(1992), 486-491.
[26] N.C. Wong, D.R. Sahu, J.C. Yao, *Solving variational inequalities involving nonexpansive type mappings*, Nonlinear Anal., **69**(2008), 4732-4753.
[27] H.K. Xu, *Inequalities in Banach spaces with applications*, Nonlinear Anal., **16**(1991), 1127-1138.
[28] H.K. Xu, *Iterative algorithms for nonlinear operators*, J. London Math. Soc., **66**(2002), 240-256.
[29] H.K. Xu, *Another control condition in an iterative method for nonexpansive mappings*, Bull. Austral. Math. Soc., **65**(2002), 109-113.
[30] H.K. Xu, *A variable Krasnoselskii-Mann algorithm and the multiple-set split feasibility problem*, Inverse Problems, **22**(2006), 2021-2034.
[31] D. Youla, *Mathematical theory of image restoration by the method of convex projections*, in: H. Stark (Ed.), Image Recovery Theory and Applications, Academic Press, Orlando, (1987), 29-77.
[32] D. Youla, *On deterministic convergence of iterations of relaxed projection operators*, J. Visual Comm. Image Representation, **1**(1990), 12-20.
[33] E. Zeidler, *Nonlinear Functional Analysis and its Applications, III: Variational Methods and Applications*, Springer, New York, NY, 1985.

Received: May 18, 2009; Accepted: March 24, 2010.