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SOME RESULTS ABOUT T-STABILITY AND ALMOST T-STABILITY

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Abstract. We shall study almost T-stability of Mann iteration for φ-contraction mappings. Also, we shall study the T-stability of Picard iteration for mappings satisfying a contractive condition of integral type.
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1. Introduction

The concept of stability of a fixed point iteration procedure seems to be due to Ostrowski, as mentioned by Rhoades [1]. It has been systematically studied by Harder in her thesis and published in the papers of Harder and Hicks ([3] and [4]). Let (X, d) be a complete metric space and $T : X \to X$ a map. Let $x_{n+1} = f(T, x_n)$ be an iteration procedure. Suppose that T has at least one fixed point and that the sequence $\{x_n\}$ converges to a fixed point $q \in X$. Let $\{y_n\}$ be an arbitrary sequence in X and define $\epsilon_n = d(y_{n+1}, f(T, y_n))$. If $\lim_{n\to\infty} \epsilon_n = 0$ implies that $\lim_{n\to\infty} y_n = q$, then the iteration procedure $x_{n+1} = f(T, x_n)$ is called T-stable and, if the convergence of the series $\sum_{i=1}^{\infty} \epsilon_i$ implies that $\lim_{n\to\infty} y_n = q$, then the iteration procedure is said to be almost T-stable.

There are some papers on T-stability of Picard iteration and equivalence between T-Stabilities of Mann, Picard and Ishikawa iterations for some mappings (see for example [5]-[12]). We shall study almost T-stability of Mann iteration for φ-contraction mappings. Also, we shall study the T-stability of Picard iteration for the mappings satisfying a contractive condition of integral type.

In this paper, we suppose that X is a normed space. Here, let us mention three iteration methods. For $x_1, u_1, s_1 \in X$, the picard iteration is given by

$$x_{n+1} = Tx_n,$$

the Mann iteration is given by

$$u_{n+1} = (1-\alpha_n)u_n + \alpha_n T u_n,$$

and the Ishikawa iteration given by

$$s_{n+1} = (1-\alpha_n)s_n + \alpha_n T t_n$$

$$t_n = (1-\beta_n)s_n + \beta_n T s_n,$$

where $\{\alpha_n\}_{n\geq 1}$ and $\{\beta_n\}_{n\geq 1}$ are sequences in $[0,1]$ and satisfy $\lim\limits_{n\to\infty} \alpha_n = 0$ and $\sum\limits_{i=1}^{\infty} \alpha_i = \infty$.

2. Some results on almost T-stability

Now, we are ready to state and prove our main results. We shall need the following preliminaries. A function $\varphi : [0,\infty) \to [0,\infty)$ is said to be a comparison function if φ is increasing, $\varphi(0) = 0$ and $\varphi(t) < t$ for all $t > 0$. Note that if φ is a comparison function, then $\varphi^n(t)$ converges to 0 for all $t > 0$.

Property (A_1)**:** We say that a mapping φ satisfies property (A_1) whenever φ is a convex comparison function and

$$\varphi(u+v) \leq u + \varphi(v)$$

for all $u, v \in [0,\infty)$.

There are many mappings which satisfy property (A_1). For example, if $g : [0,\infty) \to [0,1)$ is an increasing differentiable function, then $\varphi(t) = \int\limits_0^t g(x)dx$ satisfies property (A_1). It is clear that φ is increasing and $\varphi(0) = 0$. Since $\varphi'(t) = g(t)$ for all $t \geq 0$ and g is increasing, φ' is increasing. Hence, φ is convex. Since $1-g$ is continuous and $1-g > 0$, $\int(1-g) > 0$. Thus, $\varphi(t) < t$. Note that

$$\varphi(u+v) = \int_0^{u+v} g(x)dx = \int_0^v g(x)dx + \int_v^{u+v} g(x)dx$$

$$\leq \int_v^{u+v} 1dx + \int_0^v g(x)dx = u + \int_0^v g(x)dx = u + \varphi(v)$$

for all $u, v \geq 0$. Therefore, φ satisfies property (A_1). In particular, $\varphi(t) = t - \log(1+t)$ satisfies property (A_1).

Lemma 2.1. [1; page 13] *Let $\{t_n\}$ be a sequence in $[0,1]$ such that $\sum\limits_{n=1}^{\infty} t_n = \infty$, $\{c_n\}$ a sequence in $[0,\infty)$ such that $\sum\limits_{n=1}^{\infty} c_n < \infty$. Also, suppose that $\{a_n\}$ and $\{b_n\}$ are two sequences in $[0,\infty)$ satisfying $b_n = o(t_n)$ and $a_{n+1} \leq (1-t_n)a_n + b_n + c_n$ for all $n \geq 1$. Then, $\lim\limits_{n\to\infty} a_n = 0$.*

Theorem 2.2. *Suppose that* X *is a normed space,* $\varphi : [0, \infty) \to [0, \infty)$ *satisfies property* (A_1) *and* $T : X \to X$ *is a mapping satisfying* $F(T) = \{q\}$ *and*

$$\|Tx - q\| \leq \varphi(\|x - q\|) \tag{2.1}$$

for all $x \in X$*. Then Mann iteration is almost T-stable.*

Proof. Let $\{u_n\}_{n\geq 1}$ denote Mann iteration. Then, using (2.1) and the fact that $\varphi(t) \leq t$ for each $t \geq 0$,

$$\begin{aligned}\|u_{n+1} - q\| &= \|(1-\alpha_n)u_n + \alpha_n Tu_n - q\|\\ &\leq (1-\alpha_n)\|u_n - q\| + \alpha_n\|Tu_n - q\|\\ &\leq (1-\alpha_n)\|u_n - q\| + \alpha_n\varphi(\|u_n - q\|)\\ &\leq (1-\alpha_n)\|u_n - q\| + \alpha_n(\|u_n - q\|) = \|u_n - q\|.\end{aligned}$$

Therefore $\{\|u_n - q\|\}_{n\geq 1}$ is a nonnegative nonincreasing sequence in X and is bounded. Assume that $\sum_{n=1}^{k} y_n$ converges, where

$$y_{n+1} = \|u_{n+1} - (1-\alpha_n)u_n - \alpha_n Tu_n\|.$$

Take $M = \sup_{n\geq 1}\|u_n - q\|$. For each $\epsilon > 0$, there exists a natural number p such that

$$\sum_{i=m}^{\infty} y_i \leq \frac{\epsilon}{4},\ \varphi^m(M) < \frac{\epsilon}{4},$$

for all $m \geq p$. By considering $d_n = \|u_n - q\|$ and $c_n = \|Tu_n - q\|$ we have $d_n \leq y_n + (1-\alpha_{n-1})d_{n-1} + \alpha_{n-1}\varphi(d_{n-1})$. In fact

$$\begin{aligned}d_n = \|u_n - q\| &\leq \|u_n - (1-\alpha_{n-1})u_{n-1} - \alpha_{n-1}Tu_{n-1}\|\\ &\quad + \|(1-\alpha_{n-1})u_{n-1} + \alpha_{n-1}Tu_{n-1} - q\|\\ &= \|u_n - (1-\alpha_{n-1})u_{n-1} - \alpha_{n-1}Tu_{n-1}\|\\ &\quad + \|(1-\alpha_{n-1})u_{n-1} + \alpha_{n-1}Tu_{n-1} - (1-\alpha_{n-1})q - \alpha_{n-1}q\|\\ &\leq y_n + (1-\alpha_{n-1})\|u_{n-1} - q\| + \alpha_{n-1}\|Tu_{n-1} - q\|\\ &\leq y_n + (1-\alpha_{n-1})\|u_{n-1} - q\| + \alpha_{n-1}\varphi(\|u_{n-1} - q\|)\\ &= y_n + (1-\alpha_{n-1})d_{n-1} + \alpha_{n-1}\varphi(d_{n-1}).\end{aligned}$$

Hence,

$$(1-\alpha_n)d_n \leq (1-\alpha_n)y_n + (1-\alpha_n)(1-\alpha_{n-1})d_{n-1} + (1-\alpha_n)\alpha_{n-1}\varphi(d_{n-1}).$$

On the other hand, since φ is increasing, we have

$$\varphi(d_n) \leq \varphi(y_n + (1-\alpha_{n-1})d_{n-1} + \alpha_{n-1}\varphi(d_{n-1}).$$

If $u = y_n$ and $v = (1-\alpha_{n-1})d_{n-1} + \alpha_{n-1}\varphi(d_{n-1})$, then by using property (A_1) we have

$$\varphi(d_n) \leq y_n + \varphi((1-\alpha_{n-1})d_{n-1} + \alpha_{n-1}\varphi(d_{n-1}).$$

Since φ is a convex function, we obtain

$$\varphi(d_n) \leq y_n + (1-\alpha_{n-1})\varphi(d_{n-1}) + \alpha_{n-1}\varphi^2(d_{n-1}).$$

These relations imply that

$$\|u_{n+1}-q\| \leq y_{n+1}+(1-\alpha_n)d_n+\alpha_n\varphi(d_n)$$

$$\leq y_{n+1}+(1-\alpha_n)y_n+\alpha_n y_n+(1-\alpha_n)(1-\alpha_{n-1})d_{n-1}$$

$$+(1-\alpha_n)\alpha_{n-1}\varphi(d_{n-1})+\alpha_n(1-\alpha_{n-1})\varphi(d_{n-1})+\alpha_n\alpha_{n-1}\varphi^2(d_{n-1}).$$

Therefore, by using similar methods, we have

$$\begin{array}{rcl}
\|u_{n+1}-q\| & \leq & y_{n+1}+(1-\alpha_n)d_n+\alpha_n\varphi(d_n) \\
& \leq & y_{n+1}+(1-\alpha_n)y_n+\alpha_n y_n+(1-\alpha_n)(1-\alpha_{n-1})d_{n-1} \\
& + & (1-\alpha_n)\alpha_{n-1}\varphi(d_{n-1})+\alpha_n(1-\alpha_{n-1})\varphi(d_{n-1})+\alpha_n\alpha_{n-1}\varphi^2(d_{n-1}) \\
& \leq & y_{n+1}+y_n+(1-\alpha_n)(1-\alpha_{n-1})y_{n-1} \\
& + & (1-\alpha_n)\alpha_{n-1}y_{n-1}+\alpha_n(1-\alpha_{n-1})y_{n-1}+\alpha_n\alpha_{n-1}y_{n-1} \\
& + & (1-\alpha_n)(1-\alpha_{n-1})(1-\alpha_{n-2})d_{n-2} \\
& + & (1-\alpha_n)(1-\alpha_{n-1})\alpha_{n-2}\varphi(d_{n-2})+(1-\alpha_n)\alpha_{n-1}(1-\alpha_{n-2})\varphi(d_{n-2}) \\
& + & \alpha_n(1-\alpha_{n-1})(1-\alpha_{n-2})\varphi(d_{n-2})+(1-\alpha_n)\alpha_{n-1}\alpha_{n-2}\varphi^2(d_{n-2}) \\
& + & \alpha_n(1-\alpha_{n-1})\alpha_{n-2}\varphi^2(d_{n-2}) \\
& + & \alpha_n\alpha_{n-1}(1-\alpha_{n-2})\varphi^2(d_{n-2})+\alpha_n\alpha_{n-1}\alpha_{n-2}\varphi^3(d_{n-2}) \\
& = & y_{n+1}+y_n+y_{n-1}+(1-\alpha_n)(1-\alpha_{n-1})(1-\alpha_{n-2})d_{n-2} \\
& + & (1-\alpha_n)(1-\alpha_{n-1})\alpha_{n-2}\varphi(d_{n-2}) \\
& + & (1-\alpha_n)\alpha_{n-1}(1-\alpha_{n-2})\varphi(d_{n-2})+\alpha_n(1-\alpha_{n-1})(1-\alpha_{n-2})\varphi(d_{n-2}) \\
& + & (1-\alpha_n)\alpha_{n-1}\alpha_{n-2}\varphi^2(d_{n-2})+\alpha_n(1-\alpha_{n-1})\alpha_{n-2}\varphi^2(d_{n-2}) \\
& + & \alpha_n\alpha_{n-1}(1-\alpha_{n-2})\varphi^2(d_{n-2})+\alpha_n\alpha_{n-1}\alpha_{n-2}\varphi^3(d_{n-2}).
\end{array}$$

By continuing these replacements, after a finite number of steps, we obtain

$$\|u_{n+1}-q\| \leq \sum_{i=p+1}^{n+1} y_i+\sum_{i=0}^{n-p} S_n^i\varphi^i d_p+(\prod_{i=p}^{n}\alpha_i)\varphi^{n-p+1}d_p,$$

where S_n^i are the coefficients of $\varphi^i d_p$. Note that

$$S_n^i=\sum_{\{r_1,r_2,\ldots,r_i\}\text{ is a subset of }\{p,p+1,\ldots,n\}} \alpha_{r_1}\alpha_{r_2}\ldots\alpha_{r_i}\prod_{k\in I_i}(1-\alpha_k),$$

where $I_i=\{p,p+1,\ldots,n\}\backslash\{r_1,r_2,\ldots,r_i\}$. We show that for each $k\geq 1$, $\sum\limits_{i=0}^{k} S_k^i=1$. Clearly the equality holds when $k=1$. Assume that the equality holds for all $k\leq n$. We will show that the equality holds for $n+1$. It is clear that for each $i\leq n$ we have

$$S_{n+1}^i=(1-\alpha_{n+1})S_n^i+\alpha_{n+1}S_n^{i-1}.$$

Hence,

$$\begin{aligned}\sum_{i=0}^{n+1} S_{n+1}^i &= S_{n+1}^{n+1} + (1-\alpha_{n+1})\sum_{i=0}^{n} S_n^i + \alpha_{n+1}\sum_{i=0}^{n} S_n^{i-1} \\ &= \prod_{i=0}^{n+1}\alpha_i + (1-\alpha_{n+1} + \alpha_{n+1}(1-S_n^n) \\ &= \prod_{i=0}^{n+1}\alpha_i + (1-\alpha_{n+1}) + \alpha_{n+1} - \alpha_{n+1}\prod_{i=0}^{n}\alpha_i = 1.\end{aligned}$$

We shall now prove that $\lim_{n\to\infty} S_n^i = 0$ for all $i \geq 1$. For $i = 1$,

$$S_n^1 = \alpha_p \prod_{\substack{i=p \\ i\neq p}}^{n}(1-\alpha_i) + \alpha_{p+1}\prod_{\substack{i=p \\ i\neq p+1}}^{n}(1-\alpha_i) + \cdots + \alpha_{n+1}\prod_{\substack{i=p \\ i\neq n}}^{n}(1-\alpha_i).$$

It is clear that $S_{n+1}^1 = (1-\alpha_{n+1})S_n^1 + \alpha_{n+1}\prod_{i=p}^{n}(1-\alpha_i)$. As we know, $1-x \leq e^{-x}$ for all $x > 0$. Since $\sum_{i=p}^{\infty}\alpha_i = \infty$, $\lim_{n\to\infty}\prod_{i=p}^{n}(1-\alpha_i) = 0$ and so by lemma 2.1, $\lim_{n\to\infty} S_n^1 = 0$. Assume that $\lim_{n\to\infty} S_n^{i-1} = 0$. Then

$$S_{n+1}^i = (1-\alpha_{n+1})S_n^i + \alpha_{n+1}S_n^{i-1}.$$

By using an argument similar to case $i = 1$, by Lemma 2.1, we can deduce that $\lim_{n\to\infty} S_n^i = 0$. Let $n > 2p+1$. Since $\varphi(t) < t$ for all $t > 0$, we get

$$\begin{aligned}\|u_{n+1} - q\| &\leq \sum_{i=p+1}^{n+1} y_i + (\sum_{i=0}^{p} S_n^i)M + (\sum_{i=p+1}^{n} S_n^i)\varphi^p(M) + (\prod_{i=p}^{n}\alpha_i)\varphi^p(d_p) \\ &\leq \sum_{i=p+1}^{n+1} y_i + (\sum_{i=0}^{p} S_n^i)M + \varphi^p(M) + \varphi^p(M) \\ &< \frac{\epsilon}{4} + y_{n+1} + (\sum_{i=0}^{p} S_n^i)M + \frac{\epsilon}{4} + \frac{\epsilon}{4} \leq \epsilon + y_{n+1} + (\sum_{i=0}^{p} S_n^i)M.\end{aligned}$$

Thus, $\limsup_{n\to\infty}\|u_n - q\| \leq \epsilon$ and so $\lim_{n\to\infty}\|u_n - q\| = 0$. This shows that the Mann iteration is almost T-stable. □

Example 2.1. *Define the function $T : \mathbb{R} \to \mathbb{R}$ by $T(x) = |x| - \log(1 + |x|)$. Clearly 0 is unique fixed point of T. To show that Mann iteration is almost T-stable, by theorem 2.2, define $\varphi : [0,\infty) \to [0,\infty)$ by $\varphi(x) = x - \log(1 + x)$.*

Corollary 2.3. *Let X be a normed space, $\varphi : [0,\infty) \to [0,\infty)$ a map satisfying the property (A_1) and $T : X \to X$ a self-map satisfying*

$$\|Tx - Ty\| \leq \varphi(\max\{\|x-y\|, \frac{1}{2}[\|x-Tx\| + \|y-Ty\|], \|x-Ty\|, \|y-Tx\|\}),$$

for all $x, y \in X$. Then Mann iteration is almost T-stable.

Proof. It is known that T has a unique fixed point q. Hence, it is sufficient to show that T satisfies (2.1). If $x \neq q$, then

$$\|Tx - q\| \leq \varphi(\max\{\|x - q\|, \frac{\|x - Tx\|}{2}, \|Tx - q\|\}).$$

If $\max\{\|x - q\|, \frac{\|x-Tx\|}{2}, \|Tx - q\|\} = \|Tx - q\|$, then $\|Tx - q\| = 0$. Thus, suppose that $\|Tx - q\| \leq \varphi(\max\{\|x - q\|, \frac{\|x-Tx\|}{2}\})$. If $\|x - q\| < \frac{\|x-Tx\|}{2}$, then

$$\|Tx - q\| \leq \varphi(\frac{\|x - Tx\|}{2}) \leq \varphi(\frac{1}{2}[\|x - q\| + \|q - Tx\|]) \leq \varphi(\max\{\|x - q\|, \|q - Tx\|\}),$$

and (2.1) holds. □

Remark 2.1. *It is easy to check that our results also hold for Ishikawa iteration.*

3. T-stability of Picard iteration for maps satisfy a contractive condition of integral type

In this section, we shall verify the T-stability of Picard iteration for mappings satisfying a contractive condition of integral type. Let $\mathbb{R}_+$ be the set of nonnegative real numbers and

(i) $\psi : \mathbb{R}_+ \to \mathbb{R}_+$ is subadditive, nondecreasing and continuous from the right such that $\psi(t) < t$ for all $t > 0$;

(ii) $\varphi : \mathbb{R}_+ \to \mathbb{R}_+$ is a summable, Lebesgue-integrable and nonincreasing mapping on $(0, \infty)$ such that $\int_o^\varepsilon \varphi(t)dt > 0$ for each $\varepsilon > 0$.

Note that, there are many functions which satisfy (i). For example, suppose that $g : [0, \infty) \to [0, 1]$ is a strictly decreasing map. Then, $\psi(t) = \int_o^t g(x)dx$ satisfies (i).

Let (X, d) be a complete metric space, $\psi, \varphi : \mathbb{R}_+ \to \mathbb{R}_+$ two maps satisfying the conditions (i) and (ii) respectively and $T : X \to X$ a map satisfying the following property (A_2):

$$\int_0^{d(Tx,Ty)} \varphi(t)dt \leq \psi\left(\int_0^{M(x,y)} \varphi(t)dt\right) \quad (A_2)$$

for all $x, y \in X$, where $M(x, y) = \max\{d(x, y), d(x, Tx), d(y, Ty), d(x, Ty), d(y, Tx)\}$. From [2; Theorem 8], we can conclude that, if there exists a bounded sequence $\{y_n\}_{n\geq 0}$ with $y_{n+1} = Ty_n$ for all $n \geq 0$, then T has a unique fixed point.

Theorem 3.1. *Let (X, d) be a complete metric space and $T : X \to X$ a mapping satisfying property (A_2). If there exists a bounded sequence $\{y_n\}_{n\geq 0}$ such that $\lim_{n\to\infty} d(y_{n+1}, Ty_n) = 0$, then $\{y_n\}_{n\geq 0}$ converges to the unique fixed point of T.*

Proof. Define the sequence $\{z_n\}$ by $z_{2n} = y_n$, $z_{2n+1} = Ty_n$, and $O(z_k, n) = \{z_k, z_{k+1}, \ldots, z_{k+n}\}$. For any set A, $\delta(A)$ denotes the diameter of A. We shall show that $\delta(O(z_0))$ is finite.

Since $\lim_n d(y_{n+1}, Ty_n) = 0$, there exists a positive integer N such that, for all $n > N$, $d(y_{n+1}, Ty_n) < 1/2$.

For any $i, j > N$,

$$d(y_i, y_j) \le \delta(O(y_0)) \le M,$$

which is finite since $\{y_n\}$ is bounded.

$$d(y_i, Ty_j) \le d(y_i, y_{j+1}) + d(y_{j+1}, Ty_j) \le M + \frac{1}{2}.$$

$$d(Ty_i, Ty_j) \le d(y_{i+1}, Ty_i) + d(y_{i+1}, y_{j+1}) + d(y_{j+1}, Ty_j) \le M + 1.$$

It then follows that $\delta(O(z_0))$ is finite.

Using Lemma 7 of [2],

$$\int_0^{\delta(O(z_k,n))} \varphi(t) \le \psi^k \left(\int_0^{\delta(O(z_0))} \varphi(t)dt \right),$$

which implies that $\lim_{n,k} \delta(O(z_k, n)) = 0$. Thus $\{z_n\}$ is a Cauchy sequence, which converges to some point $q \in X$, since X is complete. It is also the case that $\lim_n y_n = \lim_n Ty_n = q$.

Using (A_2), it follows that $\lim\limits_{n\to\infty} \int_0^{\delta_n} \varphi(t)dt = 0$ and so $\lim\limits_{n\to\infty} \delta_n = 0$. Hence, $\{y_n\}_{n\ge1}$ is a Cauchy sequence. Suppose that y_n converges q. Then, we have

$$\int_0^{d(q,Tq)} \varphi(t)dt = \lim_{n\to\infty} \int_0^{d(Ty_n,Tq)} \varphi(t)dt \le \limsup_{n\to\infty} \psi \left(\int_0^{M(y_n,q)} \varphi(t)dt \right)$$

$$\le \psi \left(\int_0^{d(q,Tq)} \varphi(t)dt \right).$$

This implies that $\int_0^{d(q,Tq)} \varphi(t)dt = 0$ and so $d(q, Tq) = 0$. By a method similar to the proof of [2; Theorem 8], we can show that the fixed point of T is unique. □

Corollary 3.2. *Let (X, d) be a complete metric space and $T : X \to X$ satisfying property (A_2). If the Picard iteration is convergent, then it is T-stable.*

Letting $\psi(t) = kt$ with $k \in [0, 1)$ in (A_2), we obtain the following result.

Corollary 3.3. *Let (X, d) be a complete metric space, $k \in [0, 1)$ and $T : X \to X$ satisfying*

$$\int_0^{d(Tx,Ty)} \varphi(t)dt \le k \int_0^{M(x,y)} \varphi(t)dt, \text{ for all } x, y \in X,$$

where φ satisfies the property (ii). If there exists a bounded sequence $\{y_n\}_{n\ge1}$ such that $\lim\limits_{n\to\infty} d(y_{n+1}, Ty_n) = 0$, then the sequence $\{y_n\}_{n\ge1}$ converges to the unique fixed point T.

By considering $\varphi(t) = 1$ and $\psi(t) = kt$ with $k \in [0, 1)$ in (A_2), we obtain

Corollary 3.4. *Let (X, d) be a complete metric space, $k \in [0, 1)$ and $T : X \to X$ satisfying $d(Tx, Ty) \le k \max\{d(x, y), d(x, Tx), d(y, Ty), d(x, Ty), d(y, Tx)\}$, for all $x, y \in X$. Then the Picard iteration is T-stable.*

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