A VISCOSITY APPROXIMATION METHOD OF COMMON SOLUTIONS FOR QUASI VARIATIONAL INCLUSION AND FIXED POINT PROBLEMS

NARIN PETROT[*,1], RABIAN WANGKEEREE[*] AND POOM KUMAM[**,***,1]

*Department of Mathematics, Faculty of Science, Naresuan University
Phitsanulok 65000, Thailand.

**Department of Mathematics, Faculty of Science
King Mongkut's University of Technology Thonburi (KMUTT)
Bangmod, Thrungkru, Bangkok 10140, Thailand.

***Centre of Excellence in Mathematics, CHE
Si Ayuthaya Rd., Bangkok 10400, Thailand.
E-mail: poom.kum@kmutt.ac.th

Abstract. In this paper, we introduce a new iterative scheme for finding solutions the common element of the set of fixed points of a nonexpansive mapping and the set of solutions of the variational inclusion problem with a multivalued maximal monotone mapping and an α-inverse-strongly monotone mapping. We show that the sequence converges strongly to a common solutions for quasi variational inclusion and fixed point problems under some parameters controlling conditions. This main theorem extends a recent result of Zhang et al. [Algorithms of common solutions to quasi variational inclusion and fixed point problems. *Appl. Math. Mech.* Engl. Ed., 2008, 29(5)(2006), 571-581.] and some other authors.
Key Words and Phrases: Nonexpansive mapping, variational inequality, variational inclusion, fixed point, α-inverse-strongly monotone mapping, strictly pseudocontractive mapping.
2010 Mathematics Subject Classification: 47H05, 47J05, 47J25, 47H10.

REFERENCES

[1] R. Ahmad, Q.H. Ansari, S.S. Irfan, *Generalized variational inclusions and generalized resolvent equations in Banach spaces,* Comput. Math. Appl., **49**(2005), 18251835.
[2] F.E. Browder, W.V. Petryshyn, *Construction of fixed points of nonlinear mappings in Hilbert space*, J. Math. Anal. Appl., **20**(1967), 197-228.
[3] J. Chen, L. Zhang, T. Fan, *Viscosity approximation methods for nonexpansive mappings and monotone mappings*, J. Math. Anal. Appl., **334**(2007), 1450-1461.

[1]Corresponding author.

[4] L.C. Ceng, A. Petruşel, J.C. Yao, *Iterative approaches to solving equilibrium problems and fixed point problems of infinitely many nonexpansive mappings*, J. Optim. Theory and Appl., **143**(2009), 37-58.

[5] L.C. Ceng, A. Petruşel, J.C. Yao, *Weak convergence theorem by a modified extragradient method for nonexpansive mappings and monotone mappings*, Fixed Point Theory, **9**(2008), no. 1, 73-87.

[6] Y.J. Cho, Y. Yao, H. Zhou, *Strong convergence of an iterative algorithm for accretive operators in Banach spaces*, J. Comput. Anal. Appl., **10**(2008), no. 1, 113-125.

[7] P.L. Combettes, S.A. Hirstoaga, *Equilibrium programming using proximal-like algorithms*, Math. Program, **78**(1997), 29-41.

[8] X.P. Ding, *Perturbed proximal point for generalized quasivariational inclusions*, J. Math. Anal. Appl., **210**(1997), 88101.

[9] V.F. Demyanov, G.E. Stavroulakis, L.N. Polyakova, P.D. Panagiotopoulos, *Quasi Differentiability and Nonsmooth Modeling in Mechanics, Engineering and Economics*, Kluwer Academic, Dordrecht, 1996.

[10] Y.-P. Fang, N.-J. Huang, *Iterative algorithm for a system of variational inclusions involving H-accretive operators in Banach spaces*, Acta Math. Hungar., **108**(2005), no. 3, 183195.

[11] C.S. Hu, G. Cai, *Viscosity approximation schemes for fixed point problems and equilibrium problems and variational inequality problems*, Nonlinear Anal., **72**(2010), 1792-1808.

[12] C. Jaiboon, P. Kumam, *A hybrid extragradient viscosity approximation method for solving equilibrium problems and fixed point problems of infinitely many nonexpansive mappings*, Fixed Point Theory and Applications, **2009**(2009), Article ID 374815, 32 pages.

[13] C. Jaiboon, P. Kumam, U.W. Humphries, *Weak convergence theorem by an extragradient method for variational inequality, equilibrium and fixed point problems*, Bull. Malays. Math. Sci. Soc., **32**(2009), no. 2, 173-185.

[14] G. M. Korpelevich, *The extragradient method for finding saddle points and other problems*, Ekonomika i Matematicheskie Metody, **12**(1976), 747-756.

[15] P. Kumam, *Strong convergence theorems by an extragradient method for solving variational inequalities and equilibrium problems in a Hilbert space*, Turk. J. Math., **33**(2009), 85-98.

[16] P. Kumam, P. Katchang, *A viscosity of extragradient approximation method for finding equilibrium problems, variational inequalities and fixed point problems for nonexpansive mappings*, Nonlinear Anal. Hybrid Systems, **3**(2009), 475-486.

[17] W. Kumam, P. Kumam, *Hybrid iterative scheme by a relaxed extragradient method for solutions of equilibrium problems and a general system of variational inequalities with application to optimization*, Nonlinear Anal. Hybrid Systems, **3**(2009), 640-656.

[18] F. Liu, M.Z. Nashed, *Regularization of nonlinear Ill-posed variational inequalities and convergence rates*, Set-Valued Anal., **6**(1998), 313-344.

[19] W.B. Liu, J.E. Rubio, *Optimal shape design for systems governed by variational inequalities-2. Existence theory for the evolution case*, J. Optim. Theory Appl., **69**(1991), 373396.

[20] R. Mifflin, *Semismooth and semiconvex functions in constrained optimization*, SIAM J. Control Optim., **15**(1977), 957972.

[21] A. Moudafi, *Viscosity approximation methods for fixed-points problems*, J. Math. Anal. Appl., **241**(2000), 46-55.

[22] M.O. Osilike, D.I. Igbokwe, *Weak and strong convergence theorems for fixed points of pseudocontractions and solutions of monotone type operator equations*, Comput. Math. Appl., **40**(2000), 559-567.

[23] R.T. Rockafellar, *Monotone operators and proximal point algorithm,* SIAM J. Control Optim., **14**(1976), 877-898.

[24] T. Suzuki, *Strong convergence of krasnoselskii and manns type sequences for one-parameter nonexpansive semigroups without bochner integrals*, J. Math. Anal. Appl., **305**(2005), 227-239.

[25] S. Takahashi, W. Takahashi, *Viscosity approximation methods for equilibrium problems and fixed point problems in Hilbert spaces*, J. Math. Anal. Appl., **311**(2007), no. 1, 506-515.

[26] W. Takahashi, M. Toyoda, *Weak convergence theorems for nonexpansive mappings and monotone mappings*, J. Optim. Theory Appl. **118**(2003), 417-428.

[27] R.U. Verma, *General system of a monotone nonlinear variational inclusion problems with applications*, J. Optim. Theory Appl., **131**(2006), no. 1, 151-157.

[28] R.U. Verma, *A monotone nonlinear relaxed cocoercive variational inclusions*, Central European J. Math., **5**(2007), no. 2, 386-396.

[29] R. Wittmann, *Approximation of fixed points of nonexpansive mappings*, Arch. Math., **58**(1992), 486-491.

[30] R. Wangkeeree, U. Kamraksa, *An iterative approximation method for solving a general system of variational inequality problems and mixed equilibrium problems*, Nonlinear Anal. Hybrid Systems, **3**(2009), 615-630.

[31] Y. Yao, J.-C. Yao, *On modified iterative method for nonexpansive mappings and monotone mappings,* Appl. Math. Comput., **186**(2007), no. 2, 1551-1558.

[32] S.-S. Zhang, Joseph H.W. Lee, C.K. Chan, *Algorithms of common solutions to quasi variational inclusion and fixed point problems*, Appl. Math. Mech., Engl. Ed., **29**(2008), no. 5, 571-581.